Show/Hide Comment & Markup Toolbar

Chapter 13

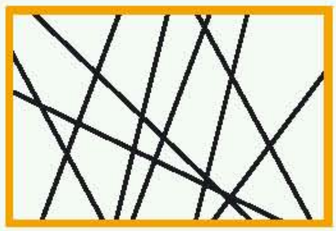
Angles in Rectilinear Figures

Learning Objectives

After completing this chapter, you will be able to

- recognize different types of angles in rectilinear figures.
- explore and use the properties of angles associated with parallel lines.
- explore and use the properties of angles of triangles.
- understand the weaknesses of intuitive approach and the strengths of deduction.
- perform simple proofs using the geometric knowledge learned.

In the figure, there are many lines crossing one another. Are there any parallel lines here?



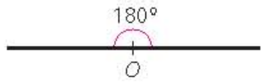
© 2009 Chung Tai Educational Press. All rights reserved.

Preview

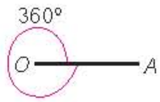
[Basic knowledge required for this chapter.]

Basic Knowledge

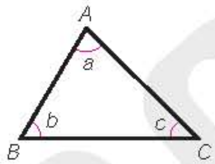
1. A straight angle is 180°.



2. A round angle is 360°.



3. The sum of angles in a triangle is 180°.
i.e. $a + b + c = 180°$.



13.1 Adjacent Angles, Angles at a Point and Vertically Opposite Angles

A Adjacent angles

In Figure 13.1, a and b are angles located on the two sides of a common arm OC with a common vertex O. a and b are called **adjacent angles**.

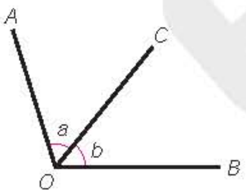
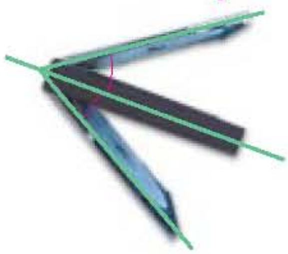


Figure 13.1



If two adjacent angles lie on the same straight line, they are called adjacent angles on a straight line. (See Figure 13.2)

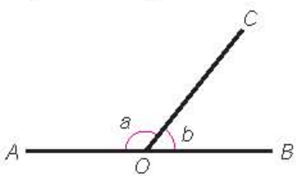


Figure 13.2

adjacent angles 鄰角

© 2009 Chung Tai Educational Press. All rights reserved.

e-Book

In Chapter 3 of S1A, we have learned that when the two arms and the vertex of an angle form a straight line, this angle is a straight angle which equals 180°. Therefore,

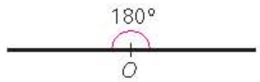
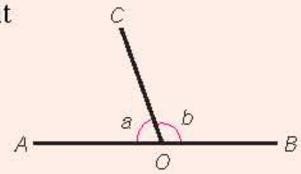


Figure 13.3

The sum of adjacent angles on a straight line is 180°.

i.e. If AOB is a straight line, then $a + b = 180°$.

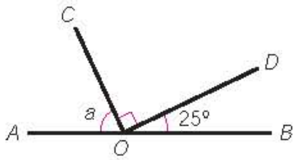
[Abbreviation: adj. ∠s on st. line]



Additional Example 13.1

Example 13.1 Finding unknowns through 'adj. ∠s on st. line'

In the figure, AOB is a straight line. Find a.



Solution

[Analysis: 'AOB is a straight line' is a given condition. According to the property of 'adj. ∠s on st. line', we know that the sum of all adjacent angles on AOB is 180°.]

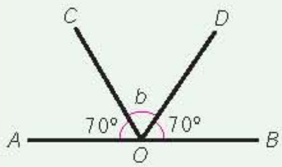
$$\begin{aligned} a+90^\circ+25^\circ &= 180^\circ \quad \text{(adj. } \angle\text{s on st. line)} \\ a+115^\circ &= 180^\circ \\ a &= 180^\circ-115^\circ \\ &= \underline{\underline{65^\circ}} \end{aligned}$$

The description inside the brackets is a reason to explain why the expression on the left of it holds.

Classwork 13.1

Solution

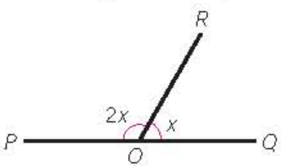
In the figure, AOB is a straight line. Find b.



Additional Example 13.2

Example 13.2 Finding unknowns through 'adj. ∠s on st. line'

In the figure, POQ is a straight line. Find x.



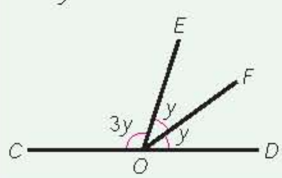
Solution

$$\begin{aligned} 2x+x &= 180^\circ \quad \text{(adj. } \angle\text{s on st. line)} \\ 3x &= 180^\circ \\ x &= \underline{\underline{60^\circ}} \end{aligned}$$

Classwork 13.2

Solution

In the figure, COD is a straight line. Find y.



e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

B Angles at a point

In Figure 13.4, p, q, r and s are four angles with a common vertex O, where p and q, q and r, r and s, s and p are four pairs of adjacent angles. p, q, r and s are called **angles at a point**.

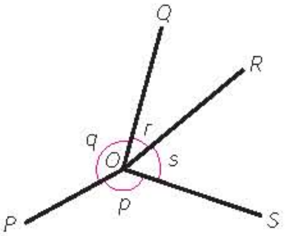
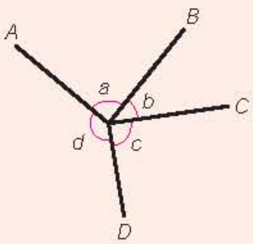


Figure 13.4

In Chapter 3 of S1A, we have learned that a round angle is 360°. From Figure 13.4, we can see angles at a point form a round angle.

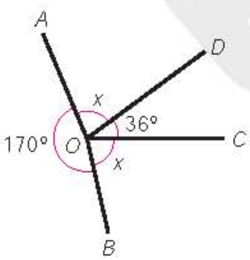
The sum of angles at a point is 360°.
i.e. If a, b, c and d are angles at a point,
then $a + b + c + d = 360°$.
[Abbreviation: ∠s at a pt.]



Additional Example 13.3

Example 13.3 Finding unknowns through '∠s at a pt.'

Find x in the figure.



Solution

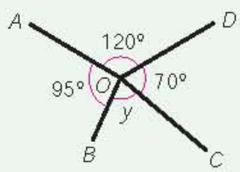
$$
\begin{aligned}
x + 36° + x + 170° &= 360° \quad (\angle\text{s at a pt.})\\
2x + 206° &= 360°\\
2x &= 360° - 206°\\
2x &= 154°\\
x &= \underline{\underline{77°}}
\end{aligned}
$$

Classwork 13.3

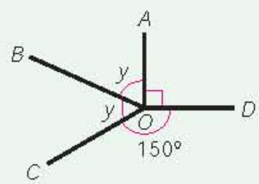
Solution

Find y in the following figures.

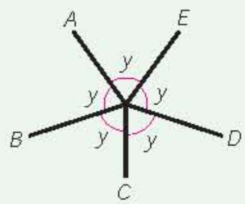
(a)



(b)



(c)



angles at a point 同頂角

© 2009 Chung Tai Educational Press. All rights reserved.

C Vertically opposite angles

In Figure 13.5, straight lines AB and CD intersect at O. a and b, x and y are called **vertically opposite angles**.

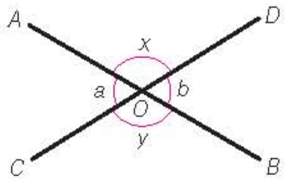


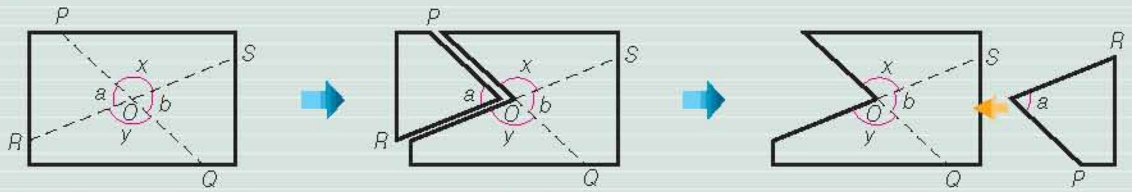
Figure 13.5

Class Activity 13.1

Aim: To explore the property of vertically opposite angles

Tools required: A piece of rectangular paper and a pair of scissors

1. (a)



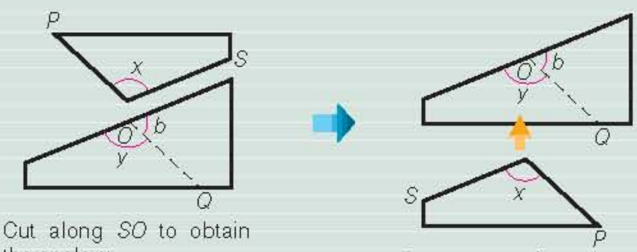
Fold a piece of rectangular paper twice such that two creases PQ and RS intersect at O. Label the two pairs of vertically opposite angles as a and b, x and y.

Cut along PO and RO to obtain the angle a.

Put a on top of b.

(b) Are the sizes of a and b the same? ☐ Yes ☐ No Show Ans.

2. (a)



Cut along SO to obtain the angle x.

Put x on top of y.

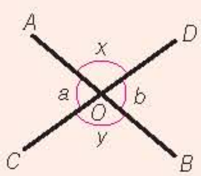
(b) Are the sizes of x and y the same? ☐ Yes ☐ No Show Ans.

From the Class Activity, we can discover the following property.

Student's CD

I.T. Tool

> Vertically opposite angles are equal.
> i.e. If AOB and COD are two straight lines,
> then $a = b$ and $x = y$.
> [Abbreviation: vert. opp. $\angle$s]



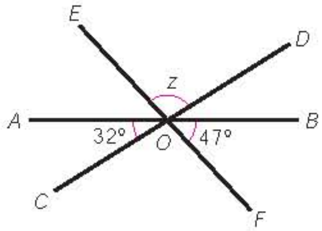
vertically opposite angles 對頂角

© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.4

Example 13.4 Finding unknowns through 'vert. opp. ∠s'

In the figure, straight lines AB, CD and EF intersect at O. Find z.



Solution

[Analysis: 'Straight lines AB, CD and EF intersect at O' is an important information, which let us
(i) make use of 'adj. ∠s on st. line' on the straight lines AOB, COD and EOF;
(ii) know that '$\angle AOC$ and $\angle BOD$', '$\angle AOE$ and $\angle BOF$' and '$\angle COF$ and $\angle DOE$' are 3 pairs of vertically opposite angles.]

$$\begin{aligned} \angle DOB &= \angle AOC \quad &\text{(vert. opp. } \angle\text{s)} \\ &= 32^\circ \end{aligned}$$

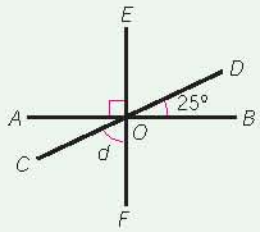
$$\begin{aligned} z + \angle DOB + 47^\circ &= 180^\circ \quad &\text{(adj. } \angle\text{s on st. line)} \\ z + 32^\circ + 47^\circ &= 180^\circ \\ z + 79^\circ &= 180^\circ \\ z &= 180^\circ - 79^\circ \\ &= \underline{\underline{101^\circ}} \end{aligned}$$

Classwork 13.4

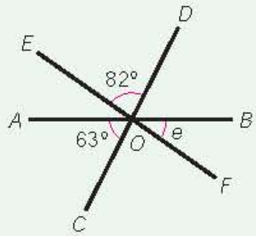
Solution

If AOB, COD and EOF are straight lines, find the unknowns in the following figures.

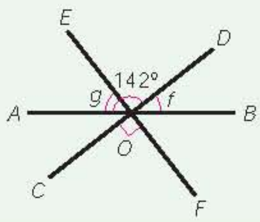
(a)



(b)



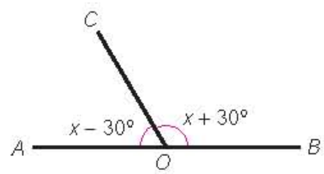
(c)



Skills Upgrading Corner 13.1

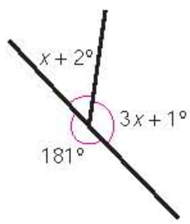
Solution

1. In the figure, AOB is a straight line.
(a) Find x.
(b) Find $\angle COB$.



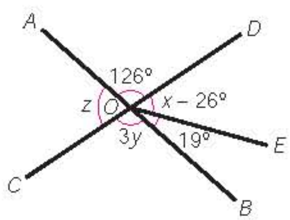
Solution

2. Find x in the figure.



Solution

3. In the figure, AOB and COD are straight lines. Find x, y and z.



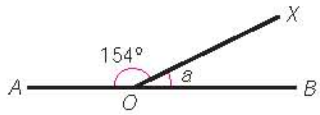
© 2009 Chung Tai Educational Press. All rights reserved.

Exercise 13A

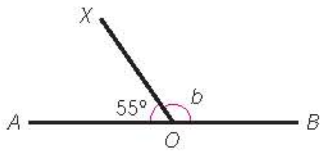
Level 1

If AOB and COD are straight lines, find the unknowns in the following figures. (**1** – **6**)

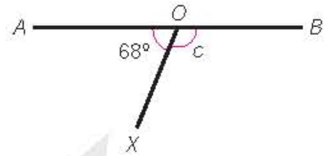
1. **(a)**



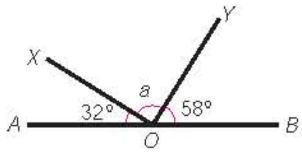
(b)



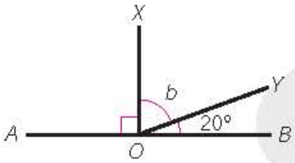
(c)



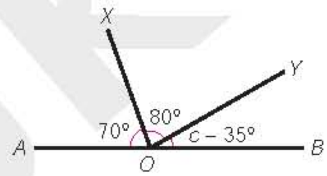
2. **(a)**



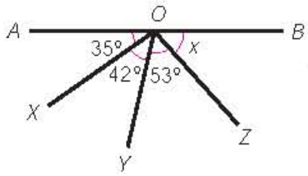
(b)



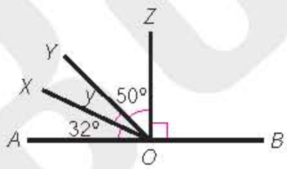
(c)



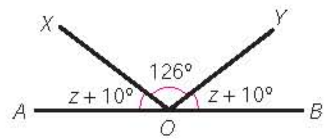
3. **(a)**



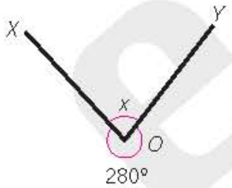
(b)



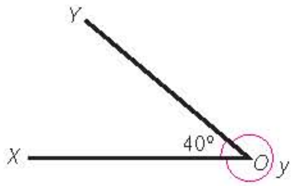
(c)



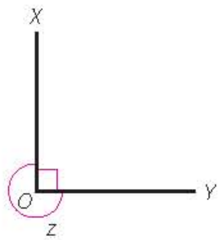
4. **(a)**



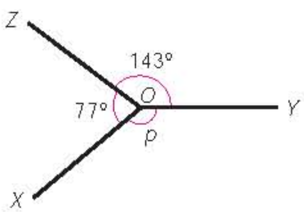
(b)



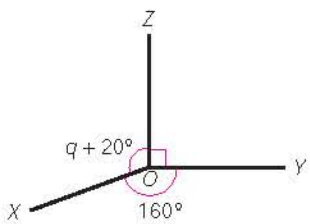
(c)



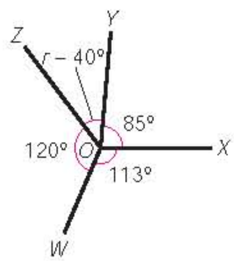
5. **(a)**



(b)

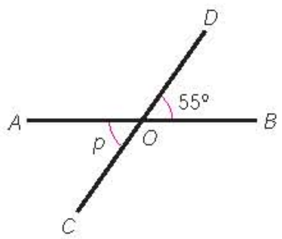


(c)

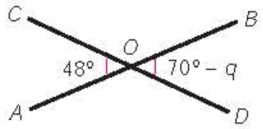


© 2009 Chung Tai Educational Press. All rights reserved.

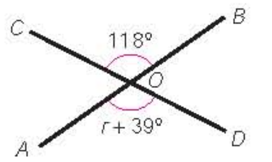
6. **(a)**



(b)



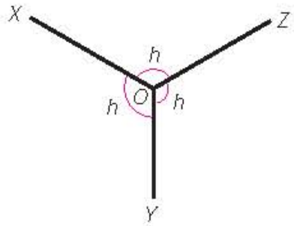
(c)



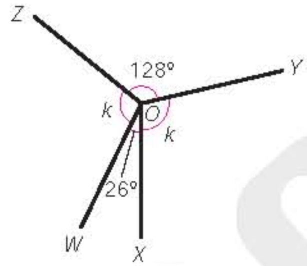
Level 2

If AOB, COD and EOF are straight lines, find the unknowns in the following figures. (**7** – **10**)

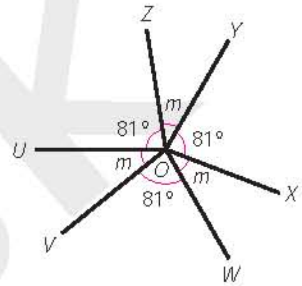
7. **(a)**



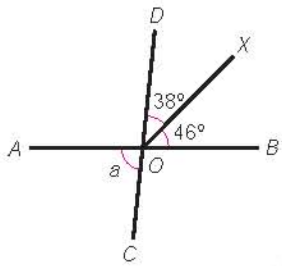
(b)



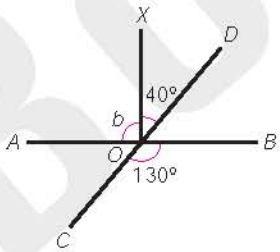
(c)



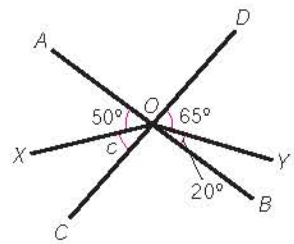
8. **(a)**



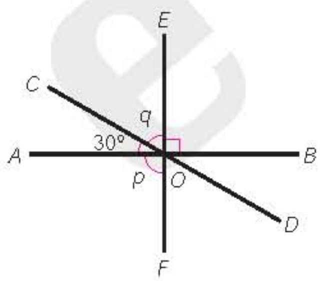
(b)



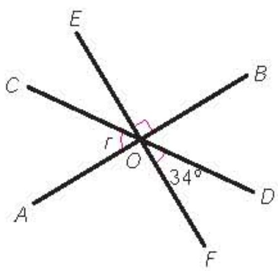
(c)



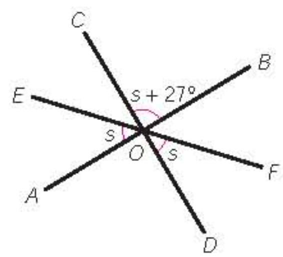
9. **(a)**



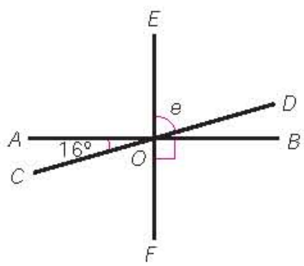
(b)



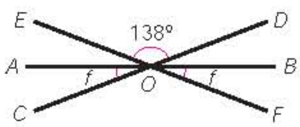
(c)



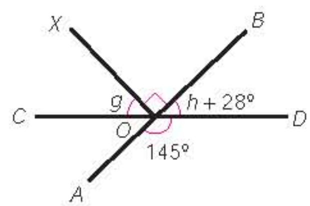
10. **(a)**



(b)



(c)



e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

13.2 Corresponding Angles, Alternate Angles, Interior Angles on the Same Side

A Transversal and angles produced by it

In Figure 13.6, a straight line XY cutting across two straight lines AB and CD, is called the **transversal**. At each intersecting point, there are four angles. Angles in pairs have different relations and names depending on their relative positions.

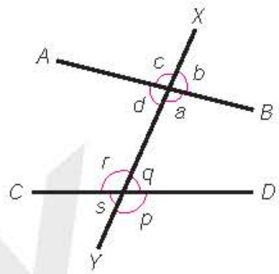


Figure 13.6

Figure 13.7 shows four pairs of **corresponding angles**.

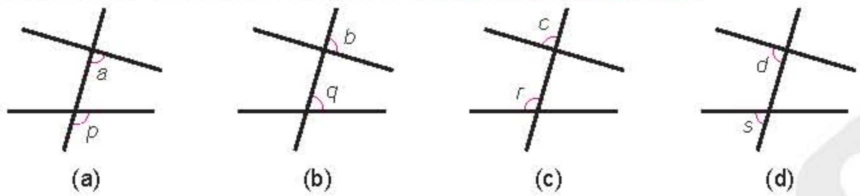


Figure 13.7

Figure 13.8 shows two pairs of **alternate angles**.

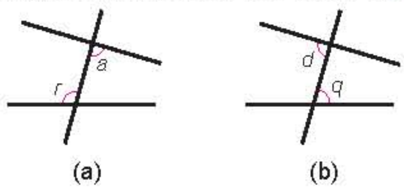


Figure 13.8

Figure 13.9 shows two pairs of **interior angles on the same side**.

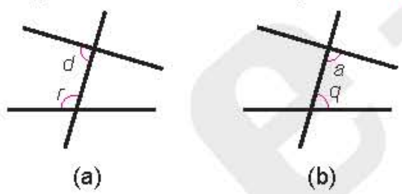


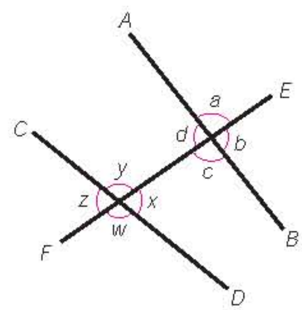
Figure 13.9

e-Book

Extension 13.1

Solution

1. In the figure, EF is a transversal cutting across AB and CD. Write down all pairs of corresponding angles, alternate angles and interior angles on the same side.



Corresponding angles: a and y, __________

Alternate angles: __________

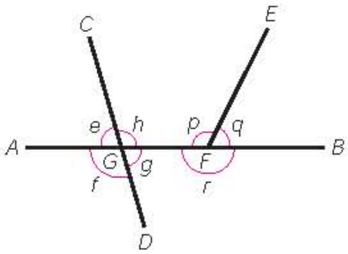
Interior angles on the same side: __________

transversal 截綫　　corresponding angles 同位角
alternate angles 錯角　　interior angles on the same side 同旁內角

© 2009 Chung Tai Educational Press. All rights reserved.

Solution

2. In the figure, $AGFB$ and CGD are straight lines. Write down all pairs of corresponding angles, alternate angles and interior angles on the same side.



Corresponding angles: ____________________

Alternate angles: ____________________

Interior angles on the same side: ____________________

B Angles associated with parallel lines

When a transversal cuts across a pair of parallel lines, corresponding angles, alternate angles and interior angles on the same side are formed.

Corresponding angles on parallel lines:

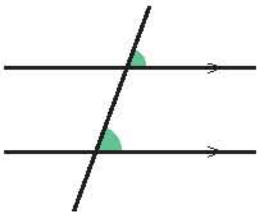


Figure 13.10

Alternate angles on parallel lines:

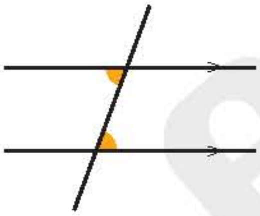


Figure 13.11

Interior angles on the same side on parallel lines:

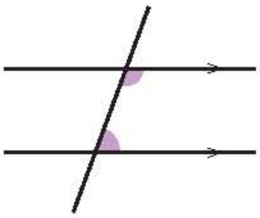


Figure 13.12

© 2009 Chung Tai Educational Press. All rights reserved.

Class Activity 13.2

Aim: To explore the properties of corresponding angles, alternate angles and interior angles on the same side on parallel lines

Tools required: A ruler, a piece of single-lined paper and a pair of scissors

1. It is known that the lines on the single-lined paper are parallel. On a piece of single-lined paper, draw two parallel lines AB and CD, and then a transversal MN cutting across the parallel lines. (See Figure I)

Show Ans.

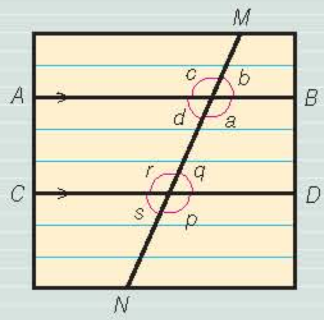


Figure I

(a) List every pair of corresponding angles in Figure I.

____

(b) List every pair of alternate angles in Figure I.

____

2. Cut the single-lined paper into four parts as shown in Figure II.

Show Ans.

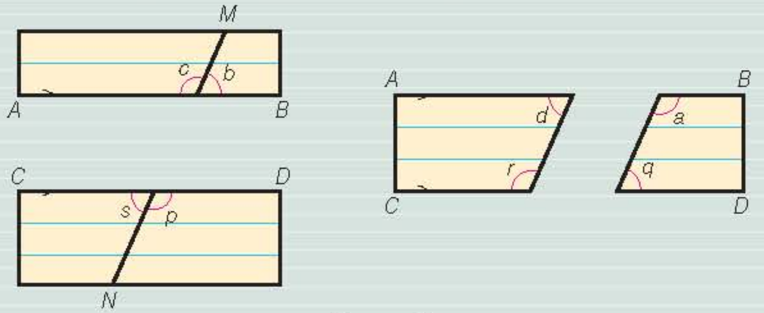


Figure II

(a) For each pair of corresponding angles, check whether the angles overlap to investigate the relation between a pair of corresponding angles on parallel lines. What do you discover?

____

(b) For each pair of alternate angles, check whether the angles overlap to investigate the relation between a pair of alternate angles on parallel lines. What do you discover?

____

3. (a) Cut the paper with angles a and q in Figure II into 2 parts and rearrange them as shown in Figure III.

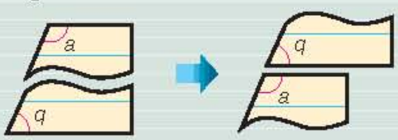


Figure III

(b) The sum of the angles a and q is ________.

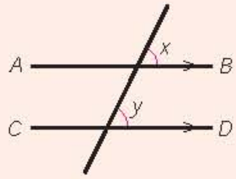
Show Ans.

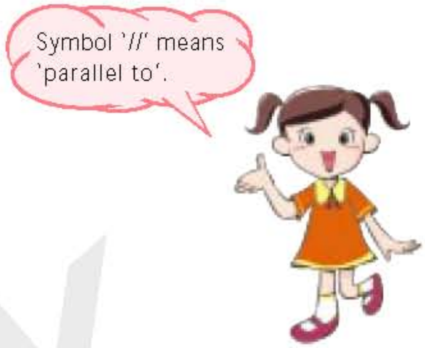
I.T. Tool

© 2009 Chung Tai Educational Press. All rights reserved.

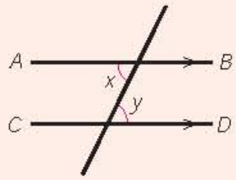
From the Class Activity, we discover the following properties.

1. If a transversal cuts a pair of parallel lines, then the corresponding angles are equal.
i.e. If $AB \parallel CD$,
then $x = y$.
[Abbreviation: corr. $\angle$s, $AB \parallel CD$]

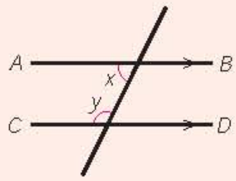




2. If a transversal cuts a pair of parallel lines, then the alternate angles are equal.
i.e. If $AB \parallel CD$,
then $x = y$.
[Abbreviation: alt. $\angle$s, $AB \parallel CD$]



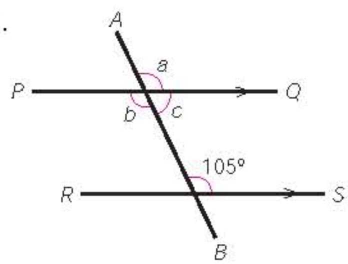
3. If a transversal cuts a pair of parallel lines, then the sum of interior angles on the same side is 180°.
i.e. If $AB \parallel CD$,
then $x + y = 180°$.
[Abbreviation: int. $\angle$s, $AB \parallel CD$]



Additional Example 13.5

Example 13.5 Finding angles on a pair of parallel lines

In the figure, $PQ \parallel RS$, AB is their transversal. Find a, b and c.



Solution

[Analysis: According to the given condition '$PQ \parallel RS$', the properties of corresponding angles, alternate angles and interior angles on the same side on parallel lines can be applied.]

$a = \underline{\underline{105°}}$ (corr. $\angle$s, $PQ \parallel RS$)

$b = \underline{\underline{105°}}$ (alt. $\angle$s, $PQ \parallel RS$)

$c + 105° = 180°$ (int. $\angle$s, $PQ \parallel RS$)

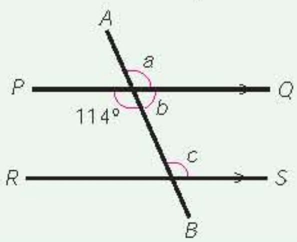
$c = 180° - 105°$

$= \underline{\underline{75°}}$

Classwork 13.5

Solution

In the figure, $PQ \parallel RS$, AB is their transversal. Find a, b and c.

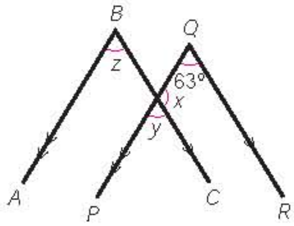


© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.6

Example 13.6 Finding angles on two pairs of parallel lines

In the figure, AB // PQ and BC // QR. Find x, y and z.



Solution

$x + 63° = 180°$ (int. $\angle$s, BC // QR)

$x = 180° - 63°$

$= \underline{\underline{117°}}$

$y = \underline{\underline{63°}}$ (corr. $\angle$s, BC // QR)

$z = y$ (corr. $\angle$s, AB // PQ)

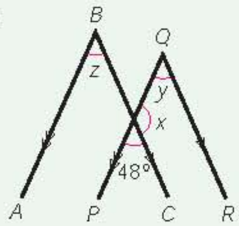
$= \underline{\underline{63°}}$

Classwork 13.6

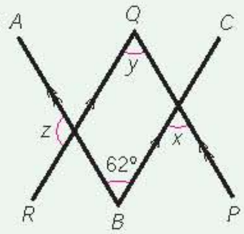
Solution

In each of the following figures, AB // PQ and BC // QR. Find x, y and z.

(a)



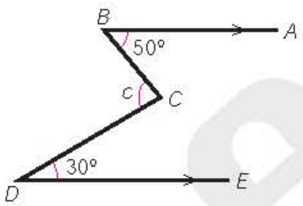
(b)



Additional Example 13.7

Example 13.7 Finding angles by constructing lines and properties of parallel lines

Find c in the figure.

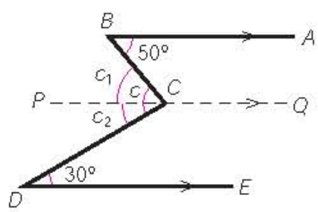


Solution

[Analysis: There is no transversal cutting the two parallel lines BA and DE in the figure. We can draw a line through C and parallel to BA and DE so that BC and CD become transversals, and hence apply the properties of parallel lines.]

Draw a straight line PCQ which is parallel to BA and DE.

Let $\angle BCP = c_1$ and $\angle DCP = c_2$.



$c_1 = 50°$ (alt. $\angle$s, BA // PQ)

$c_2 = 30°$ (alt. $\angle$s, DE // PQ)

$c = c_1 + c_2$

$= 50° + 30°$

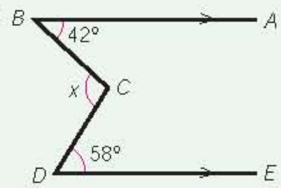
$= \underline{\underline{80°}}$

Classwork 13.7

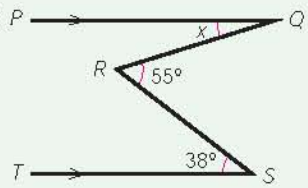
Solution

Find x in each of the following figures.

(a)



(b)

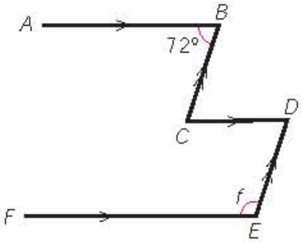


© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.8

Example 13.8 Finding angles on two pairs of parallel lines

In the figure, find f.



Solution

Let $\angle BCD = x$ and $\angle CDE = y$.

$x = 72°$ (alt. $\angle$s, AB // CD)

$y = x$ (alt. $\angle$s, BC // DE)

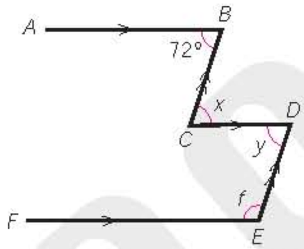
$= 72°$

$f + y = 180°$ (int. $\angle$s, CD // FE)

$f = 180° - y$

$= 180° - 72°$

$= \underline{\underline{108°}}$



Alternative method:

Produce AB and ED to intersect at G.

Let $\angle BGD = g$.

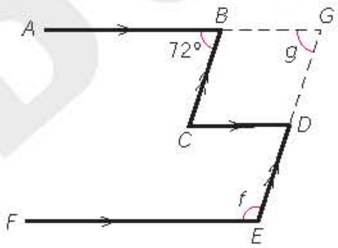
$g = 72°$ (corr. $\angle$s, BC // GE)

$g + f = 180°$ (int. $\angle$s, AG // FE)

$72° + f = 180°$

$f = 180° - 72°$

$= \underline{\underline{108°}}$

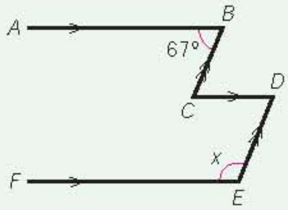


Classwork 13.8

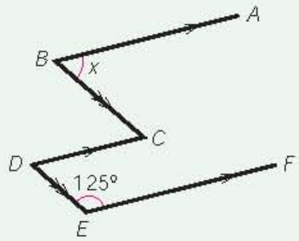
Solution

Find x in each of the following figures.

(a)



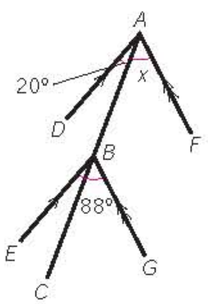
(b)



Skills Upgrading Corner 13.2

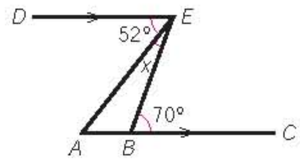
Solution

1. In the figure, ABC is a straight line, AD // BE and AF // BG. Find x.

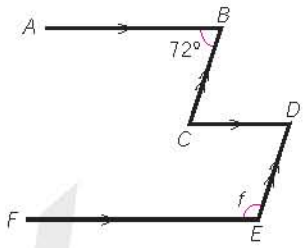


© 2009 Chung Tai Educational Press. All rights reserved.

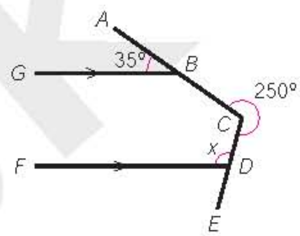
Solution **2.** In the figure, ABC is a straight line, AC // DE. Find x.



Solution **3.** In Example 13.8, we can also find f by producing BC to intersect EF. Find f in Example 13.8 under this method.



Solution **4.** In the figure, BG // DF, ABC and CDE are straight lines. Find x.

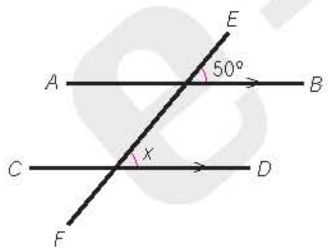


Exercise 13B

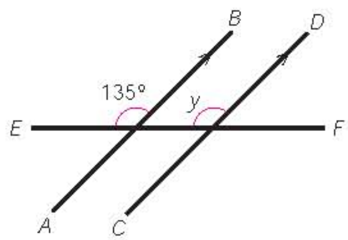
Level 1

1. In each of the following figures, AB // CD, EF is their transversal. Find the unknowns.

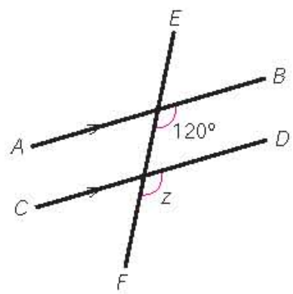
(a)



(b)

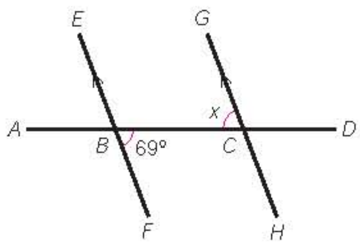


(c)

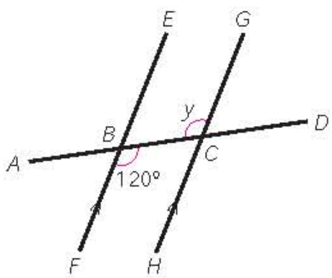


2. In each of the following figures, EF // GH, AD intersects EF and GH at B and C respectively. Find the unknowns.

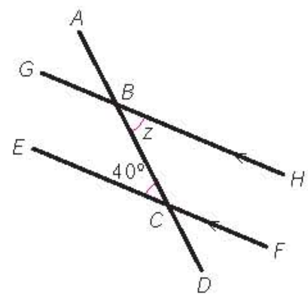
(a)



(b)



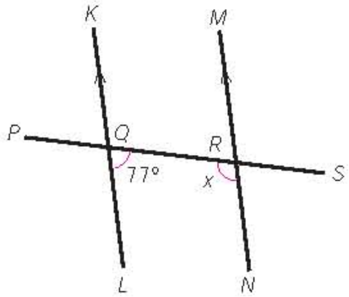
(c)



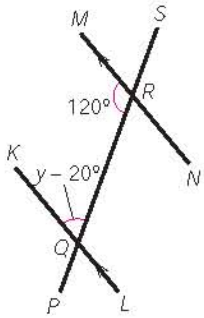
© 2009 Chung Tai Educational Press. All rights reserved.

3. In each of the following figures, KL // MN, PS intersects KL and MN at Q and R respectively. Find the unknowns.

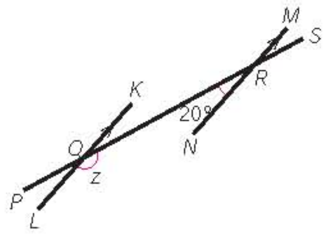
(a)



(b)

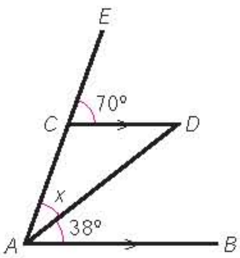


(c)

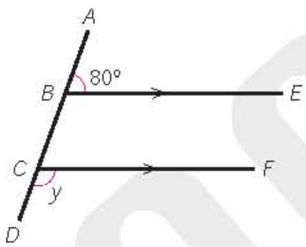


Find the unknowns in each of the following figures. (**4** – **7**)

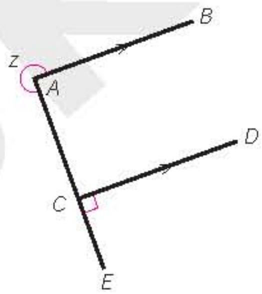
4. (a) ACE is a straight line.



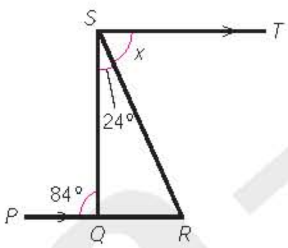
(b) $ABCD$ is a straight line.



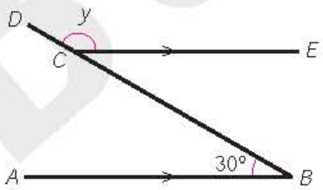
(c) ACE is a straight line.



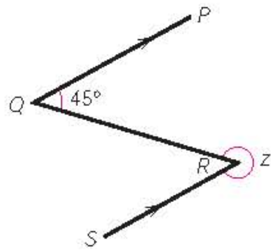
5. (a) PQR is a straight line.



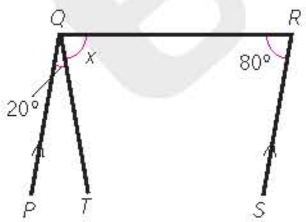
(b) BCD is a straight line.



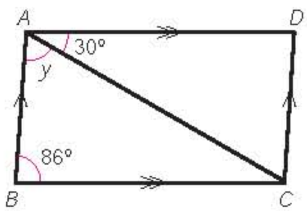
(c)



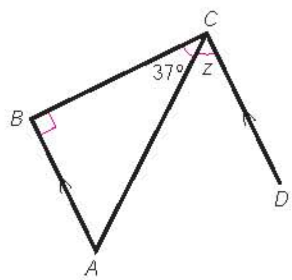
6. (a)



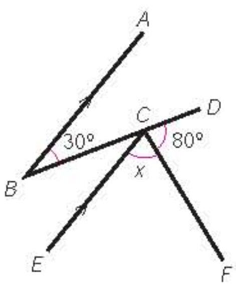
(b)



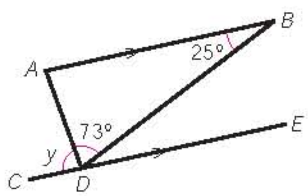
(c)



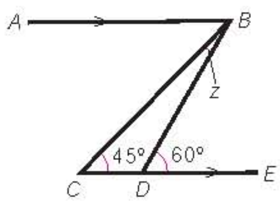
7. (a) BCD is a straight line.



(b) CDE is a straight line.



(c) CDE is a straight line.



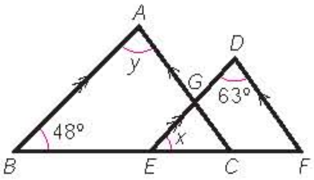
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

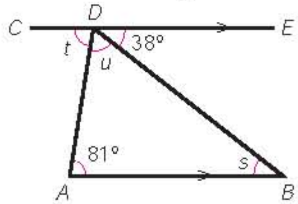
Level 2

Find the unknowns in each of the following figures. (**8** – **11**)

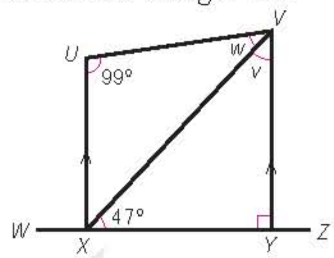
8. **(a)** *BECF*, *AGC* and *DGE* are straight lines.



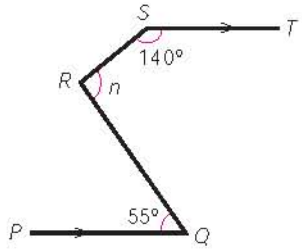
(b) *CDE* is a straight line.



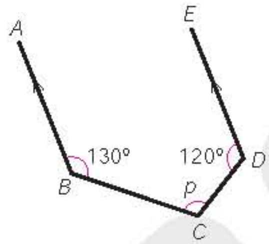
(c) *WXYZ* is a straight line.



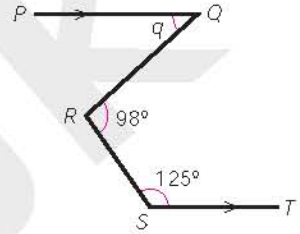
9. **(a)**



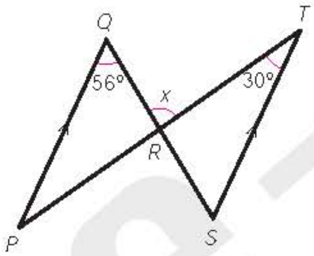
(b)



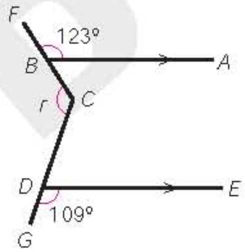
(c)



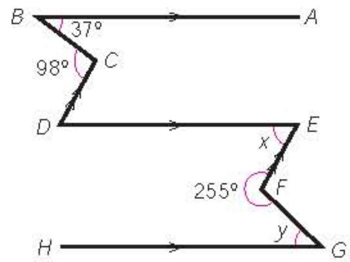
10. **(a)** *PRT* and *QRS* are straight lines.



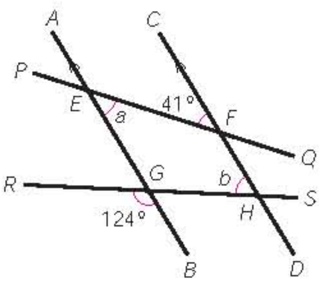
(b) *FBC* and *CDG* are straight lines.



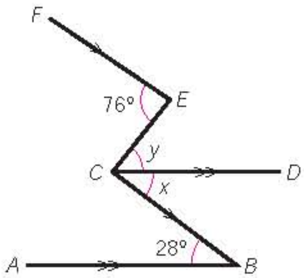
(c)



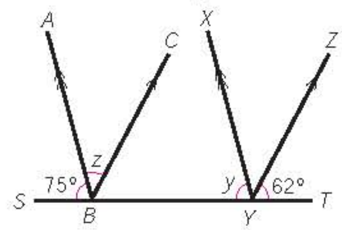
11. **(a)** *AEGB*, *CFHD*, *PEFQ* and *RGHS* are straight lines.



(b)



(c) *SBYT* is a straight line.



e-Book

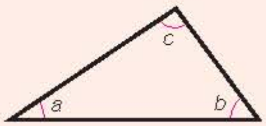
© 2009 Chung Tai Educational Press. All rights reserved.

13.3 Angles of Triangles

A Sum of the interior angles of a triangle

In Chapter 3 of S1A, we have learned that the sum of all interior angles of a triangle is 180°.

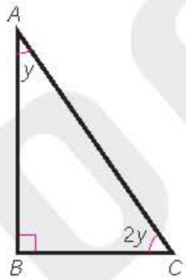
The sum of all interior angles of a triangle is 180°.
i.e. $a + b + c = 180°$.
[Abbreviation: ∠ sum of Δ]



Additional Example 13.9

Example 13.9 Finding unknowns through '∠ sum of Δ'

Find y in the figure.



Solution

In ΔABC,

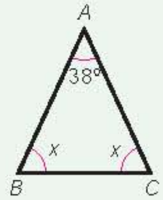
$$\begin{aligned} y + 90° + 2y &= 180° \quad (\angle \text{ sum of } \Delta) \\ 3y + 90° &= 180° \\ 3y &= 180° - 90° \\ 3y &= 90° \\ y &= \underline{\underline{30°}} \end{aligned}$$

Classwork 13.9

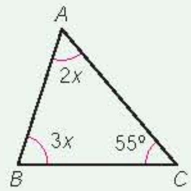
Solution

Find x in each of the following figures.

(a)



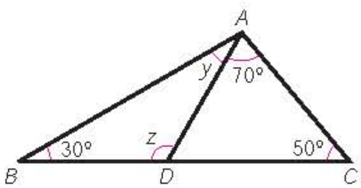
(b)



Additional Example 13.10

Example 13.10 Finding angles in several triangles

In the figure, BDC is a straight line.
Find y and z.



Solution

In ΔABC,

$$\begin{aligned} 30° + 50° + 70° + y &= 180° \quad (\angle \text{ sum of } \Delta) \\ 150° + y &= 180° \\ y &= 180° - 150° \\ &= \underline{\underline{30°}} \end{aligned}$$

In ΔABD,

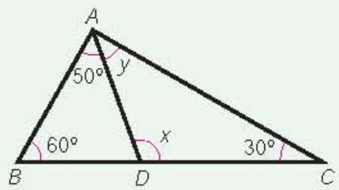
$$\begin{aligned} 30° + y + z &= 180° \quad (\angle \text{ sum of } \Delta) \\ 30° + 30° + z &= 180° \\ z &= 180° - 60° \\ &= \underline{\underline{120°}} \end{aligned}$$

Classwork 13.10

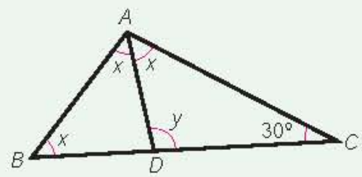
Solution

In each of the following figures, BDC is a straight line. Find x and y.

(a)



(b)



© 2009 Chung Tai Educational Press. All rights reserved.

B Exterior angles of triangles

In Figure 13.13, the side BC of ΔABC is produced to D. d is called the **exterior angle** of ΔABC, a and b are called the **interior opposite angles** to d.

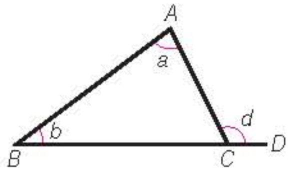
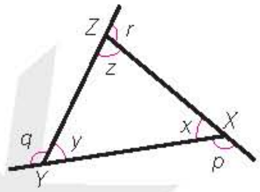


Figure 13.13

Extension 13.2

Solution

In the figure, p, q and r are the exterior angles of ΔXYZ. Write down their corresponding interior opposite angles.



(a) ________ and ________ are the interior opposite angles to p.

(b) ________ and ________ are the interior opposite angles to q.

(c) ________ and ________ are the interior opposite angles to r.

Class Activity 13.3

Aim: To explore the property of exterior angles of triangles

Tools required: A piece of triangular paper and a ruler

1. Label the interior angles as a, b and c of the triangular paper. Place it along the side of a ruler. (See Figure I)

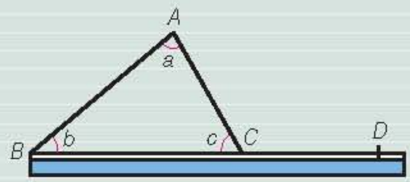


Figure I

2. Tear interior angles a and b from the paper and put them next to c. (See Figure II)

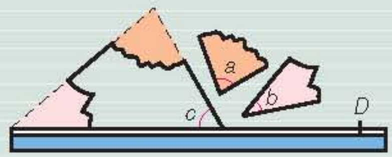


Figure II

3. What is the relation between the exterior angle $\angle ACD$ and the two interior opposite angles a and b?

Show Ans.

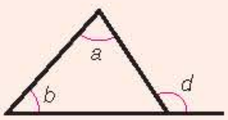
____

From the Class Activity, we discover the following property.

The exterior angle of a triangle is equal to the sum of its two interior opposite angles.

i.e. $d = a + b$.

[Abbreviation: ext. $\angle$ of Δ]



I.T. Tool

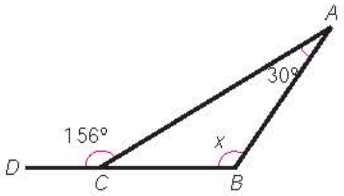
exterior angle 外角 interior opposite angle 內對角

© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.11

Example 13.11 Finding unknowns through ‘ext. ∠ of Δ’

In the figure, BCD is a straight line. Find x.



Solution

[Analysis: According to the given condition ‘BCD is a straight line’, we know that $\angle ACD$ is an exterior angle of ΔABC, $\angle CAB$ and $\angle ABC$ are interior opposite angles to $\angle ACD$.]

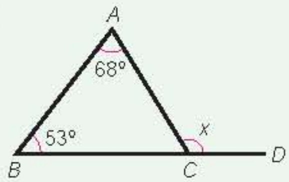
$$\begin{aligned} x + 30° &= 156° \quad \text{(ext. } \angle \text{ of } \Delta) \\ x &= 156° - 30° \\ &= \underline{\underline{126°}} \end{aligned}$$

Classwork 13.11

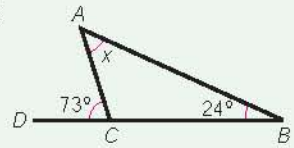
Solution

In each of the following figures, BCD is a straight line. Find x.

(a)



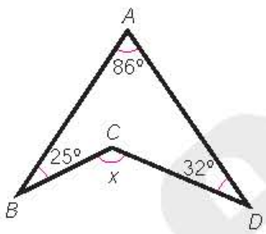
(b)



Additional Example 13.12

Example 13.12 Finding angles in a polygon by constructing lines

In the figure, find x.



Solution

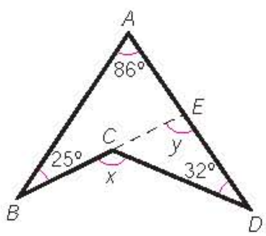
[Analysis: $ABCD$ is a quadrilateral. Although the properties of a quadrilateral have not yet been learned, $ABCD$ can be cut into two triangles for solving the problem.]

Produce BC to meet AD at E.

Let $\angle CED = y$.

In ΔABE,

$$\begin{aligned} y &= 25° + 86° \quad \text{(ext. } \angle \text{ of } \Delta) \\ &= 111° \end{aligned}$$

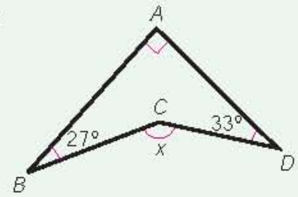


Classwork 13.12

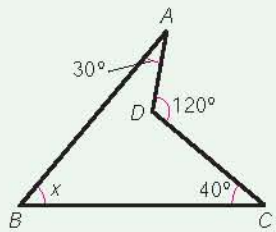
Solution

Find x in each of the following figures.

(a)



(b)



© 2009 Chung Tai Educational Press. All rights reserved.

In ΔCDE,

$x = y + 32^\circ$ (ext. $\angle$ of Δ)

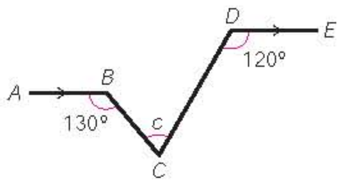
$= 111^\circ + 32^\circ$

$= \underline{\underline{143^\circ}}$

Additional Example 13.13

Example 13.13 Finding angles through the properties of parallel lines and triangles

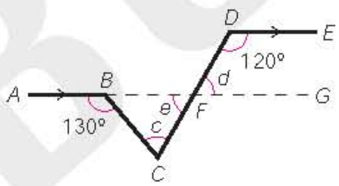
In the figure, find c.



Solution

[Analysis: There is no transversal cutting the parallel lines *AB* and *DE*. We can produce *AB* and to cut *DC* so that *DC* becomes a transversal of *DE* and *AB* produced, and hence apply the properties of parallel lines.]

Produce AB to meet CD at F.



$d + 120^\circ = 180^\circ$ (int. $\angle$s, $DE \,//\, AG$)

$d = 180^\circ - 120^\circ$

$= 60^\circ$

$e = d$ (vert. opp. $\angle$s)

$= 60^\circ$

$c + e = 130^\circ$ (ext. $\angle$ of Δ)

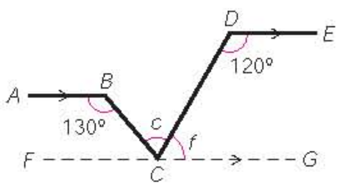
$c + 60^\circ = 130^\circ$

$c = 130^\circ - 60^\circ$

$= \underline{\underline{70^\circ}}$

Alternative method:

Draw a straight line FCG parallel to AB and DE.



$f + 120^\circ = 180^\circ$ (int. $\angle$s, $DE \,//\, FG$)

$f = 60^\circ$

$c + f = 130^\circ$ (alt. $\angle$s, $AB \,//\, FG$)

$c + 60^\circ = 130^\circ$

$c = 130^\circ - 60^\circ$

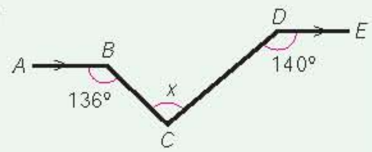
$= \underline{\underline{70^\circ}}$

Classwork 13.13

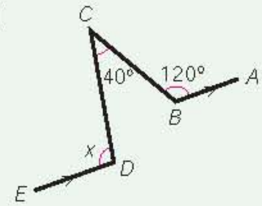
Solution

Find x in each of the following figures.

(a)



(b)



© 2009 Chung Tai Educational Press. All rights reserved.

C Base angles of isosceles triangles

Class Activity 13.4

Aim: To explore the base angles of an isosceles triangle

Tool required: A protractor

1. Figure I shows ΔXYZ. Show Ans.

(a) According to the lengths of sides of the triangle,

XYZ is __________________ triangle.

(b) Use a protractor to measure all interior angles of ΔXYZ.

$\angle X =$ ____________ , $\angle Y =$ ____________ , $\angle Z =$ ____________

(c) $\angle Y$ and $\angle Z$ are called the **base angles** of ΔXYZ. What is the relation between them?

____

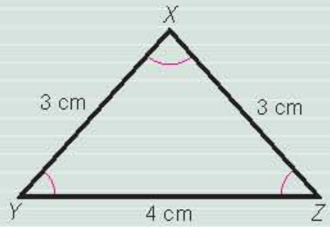


Figure I

2. Figure II shows ΔABC. Show Ans.

(a) According to the lengths of sides of the triangle,

ABC is __________________ triangle.

(b) Use a protractor to measure all interior angles of ΔABC.

$\angle A =$ ____________ , $\angle B =$ ____________ , $\angle C =$ ____________

(c) What is the relation among the three interior angles of ΔABC?

____

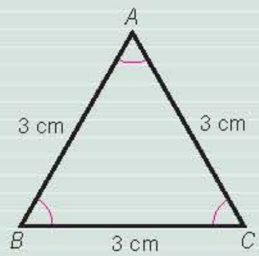
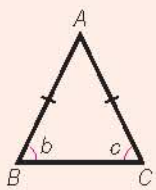


Figure II

From the Class Activity, we discover the following property.

I.T. Tool

The base angles of an isosceles triangle are equal.
i.e. If $AB = AC$,
then $b = c$.
[**Abbreviation:** base $\angle$s, isos. Δ]



From the above property, it is easy to see that all interior angles of an equilateral triangle are the same, i.e. $a = b = c$.

$\therefore$ Each interior angle of an equilateral triangle $= \dfrac{180^\circ}{3}$

$= 60^\circ$

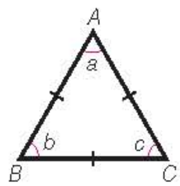


Figure 13.14

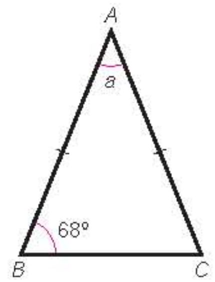
base angles 底角

© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.14

Example 13.14 Finding angles through 'base $\angle$s, isos. Δ' and '$\angle$ sum of Δ'

In the figure, find a.



Solution

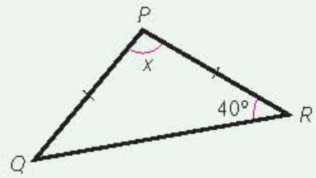
$$\angle ACB = \angle ABC \qquad \text{(base } \angle\text{s, isos. } \Delta\text{)}$$
$$= 68^\circ$$
$$a + 68^\circ + 68^\circ = 180^\circ \qquad (\angle \text{ sum of } \Delta)$$
$$a + 136^\circ = 180^\circ$$
$$a = 180^\circ - 136^\circ$$
$$= \underline{\underline{44^\circ}}$$

Classwork 13.14

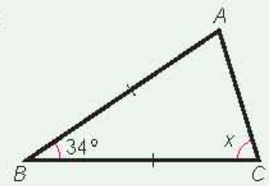
Solution

Find x in each of the following figures.

(a)



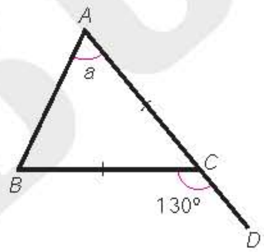
(b)



Additional Example 13.15

Example 13.15 Finding unknowns through 'base $\angle$s, isos. Δ' and 'ext. $\angle$ of Δ'

In the figure, ACD is a straight line. Find a.



Solution

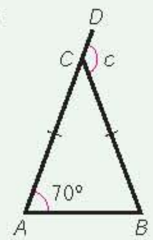
$$\angle ABC = \angle BAC \qquad \text{(base } \angle\text{s, isos. } \Delta\text{)}$$
$$= a$$
$$a + a = 130^\circ \qquad \text{(ext. } \angle \text{ of } \Delta\text{)}$$
$$2a = 130^\circ$$
$$a = \underline{\underline{65^\circ}}$$

Classwork 13.15

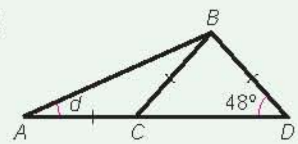
Solution

In each of the following figures, ACD is a straight line. Find the unknowns.

(a)



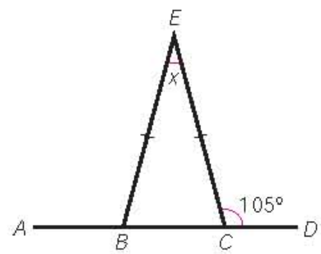
(b)



Skills Upgrading Corner 13.3

Solution

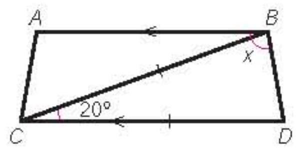
1. In the figure, $ABCD$ is a straight line, $EB = EC$. Find x.



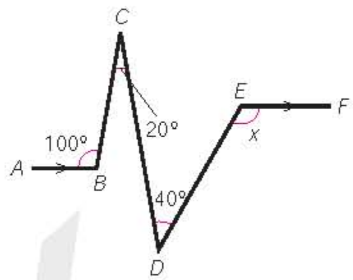
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

Solution **2.** In the figure, AB // CD, $CB = CD$. Find x.



Solution **3.** In the figure, AB // EF. Find x.

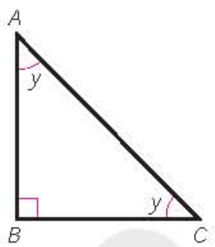


Exercise 13C

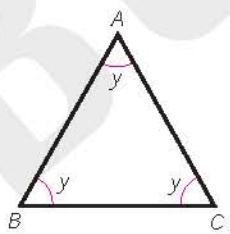
Level 1

Find the unknowns in each of the following figures. (**1** – **7**)

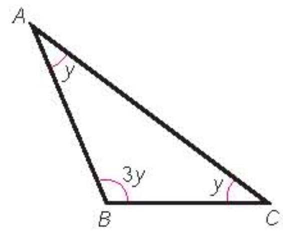
1. **(a)**



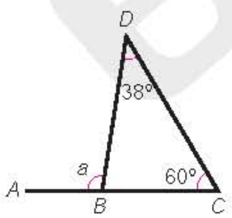
(b)



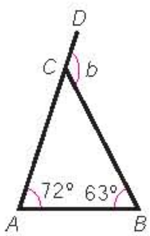
(c)



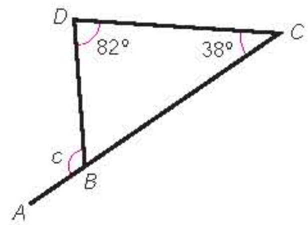
2. **(a)** ABC is a straight line.



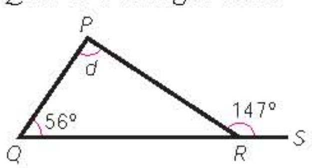
(b) ACD is a straight line.



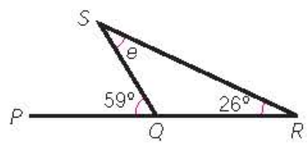
(c) ABC is a straight line.



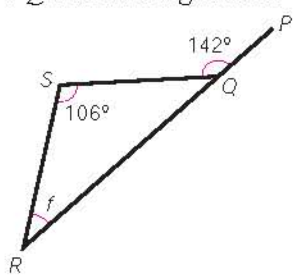
3. **(a)** QRS is a straight line.



(b) PQR is a straight line.



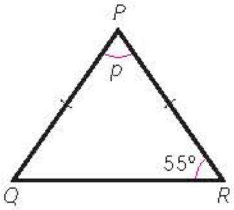
(c) PQR is a straight line.



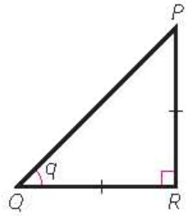
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

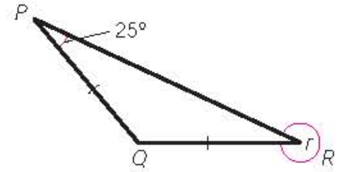
4. **(a)**



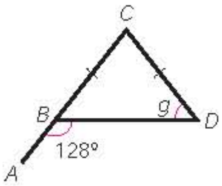
(b)



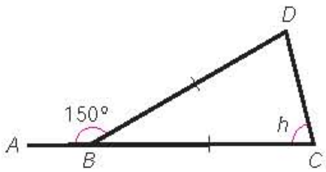
(c)



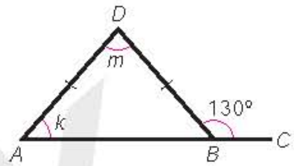
5. **(a)** ABC is a straight line.



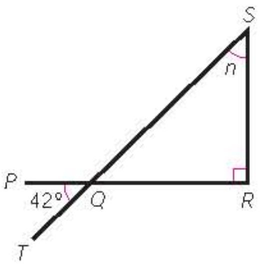
(b) ABC is a straight line.



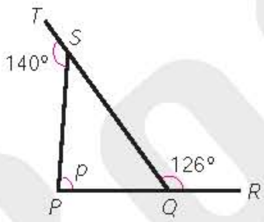
(c) ABC is a straight line.



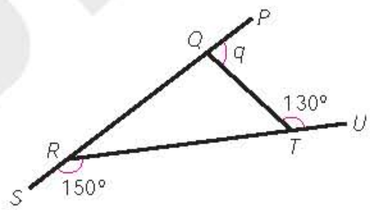
6. **(a)** PQR and TQS are straight lines.



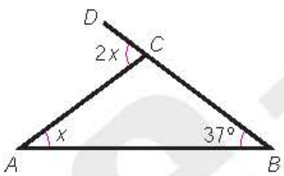
(b) PQR and QST are straight lines.



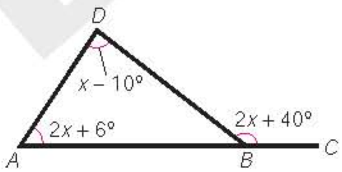
(c) RTU and $PQRS$ are straight lines.



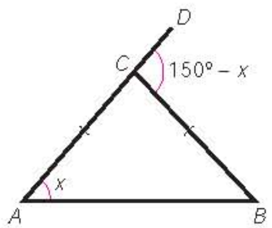
7. **(a)** BCD is a straight line.



(b) ABC is a straight line.



(c) ACD is a straight line.



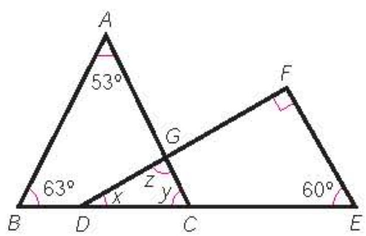
Level 2

8. In the figure, $BDCE$, AGC and DGF are straight lines.

(a) Consider ΔDEF, find x.

(b) Find y.

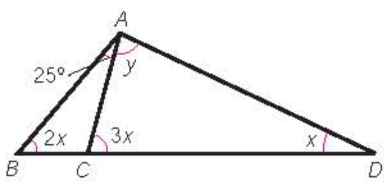
(c) Find z.



9. In the figure, BCD is a straight line.

(a) Consider ΔABC, find x.

(b) Find y.

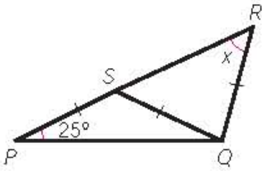


e-Book

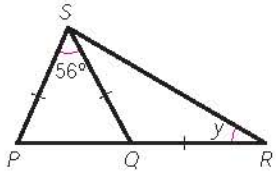
© 2009 Chung Tai Educational Press. All rights reserved.

Find the unknown in each of the following figures. (**10** – **15**)

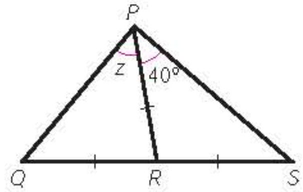
10. **(a)** *PSR* is a straight line.



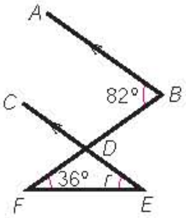
(b) *PQR* is a straight line.



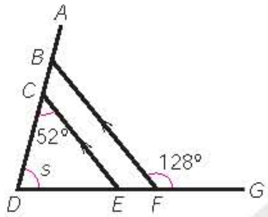
(c) *QRS* is a straight line.



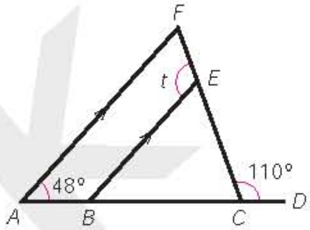
11. **(a)** *BDF* and *CDE* are straight lines.



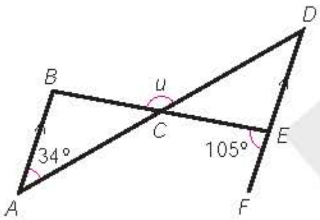
(b) *ABCD* and *DEFG* are straight lines.



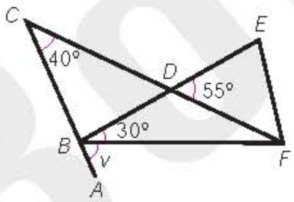
(c) *ABCD* and *CEF* are straight lines.



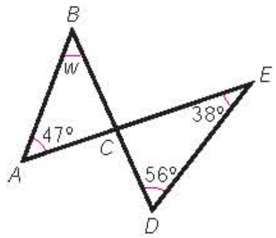
12. **(a)** *DEF*, *ACD* and *BCE* are straight lines.



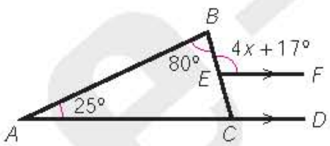
(b) *ABC*, *BDE* and *CDF* are straight lines.



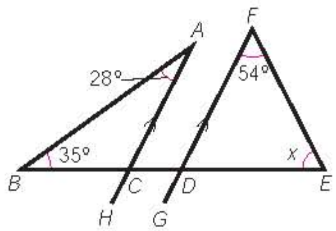
(c) *BCD* and *ACE* are straight lines.



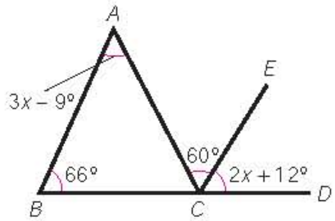
13. **(a)** *ACD* and *BEC* are straight lines.



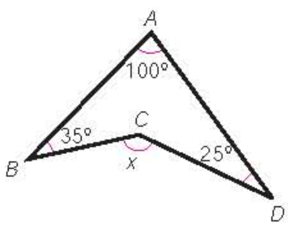
(b) *ACH*, *FDG* and *BCDE* are straight lines.



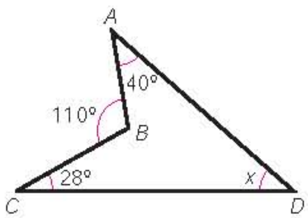
(c) *BCD* is a straight line.



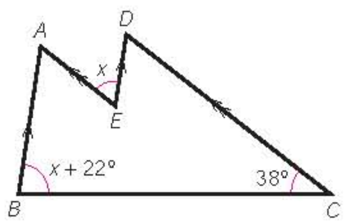
14. **(a)**



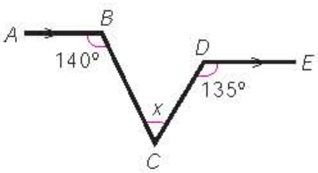
(b)



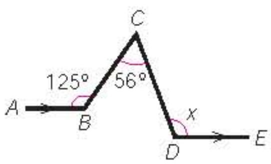
(c)



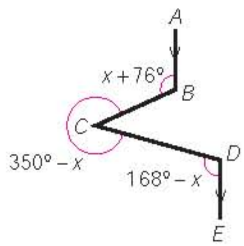
15. **(a)**



(b)



(c)



© 2009 Chung Tai Educational Press. All rights reserved.

13.4 Simple Proofs in Geometry

A Are your observation and experience reliable?

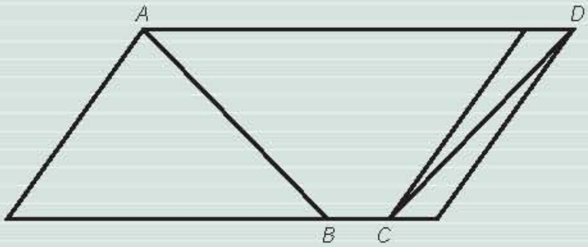
Class Activity 13.5

Aim: To explore the weaknesses of 'judgement by observation'

Tools required: A ruler and set squares

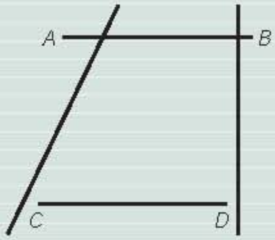
1. (a) Observe the following figures. Compare the lengths of AB and CD in each figure without using any tools, and fill in the blanks with '<', '=' or '>'.

(i)



AB ________ CD

(ii)



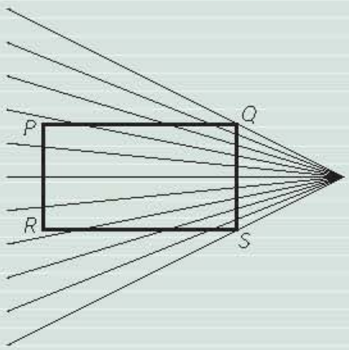
AB ________ CD

(b) Use a ruler to measure the above line segments. Is your observation correct?

☐ Yes ☐ No

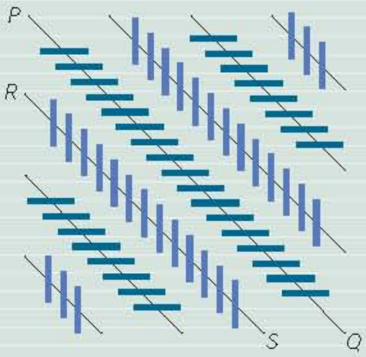
2. (a) Observe the following figures. Do PQ and RS look parallel to each other?

(i)



PQ and RS look parallel / not parallel to each other.

(ii)



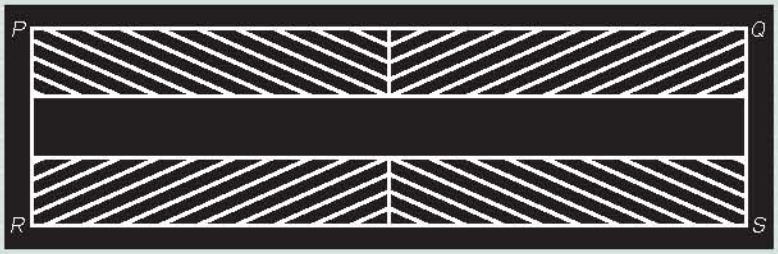
PQ and RS look parallel / not parallel to each other.

(b) Use a ruler and set squares to determine whether each pair of lines are parallel to each other. Is your observation correct?

☐ Yes ☐ No

© 2009 Chung Tai Educational Press. All rights reserved.

3. (a) Observe the following figure. Do PQ and RS look like straight lines?



☐ Yes ☐ No

(b) Use a ruler to check the above line segments. Is your observation correct?

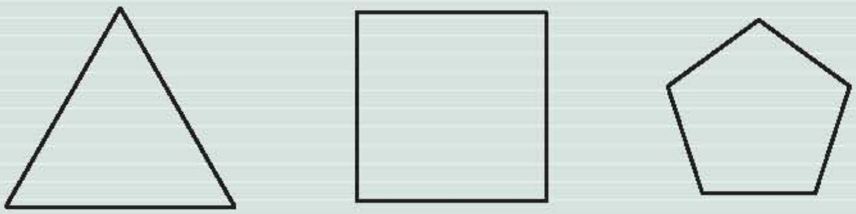
☐ Yes ☐ No

Class Activity 13.6

Aim: To explore the weaknesses of 'judgement by experience'

Tools required: A ruler and a protractor

Measure the sides and the interior angles of the following polygons.



1. For each of the above polygons, are the lengths of all sides equal? Show Ans.

____

2. For each of the above polygons, are the sizes of all interior angles equal? Show Ans.

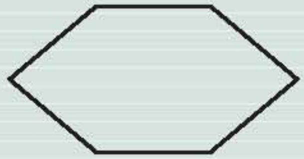
____

3. From the experience above, Joseph concludes, 'All interior angles of an equilateral polygon are equal'. Is his judgement correct? Show Ans.

☐ Yes ☐ No

© 2009 Chung Tai Educational Press. All rights reserved.

4. Zoe draws a polygon as shown. Show Ans.

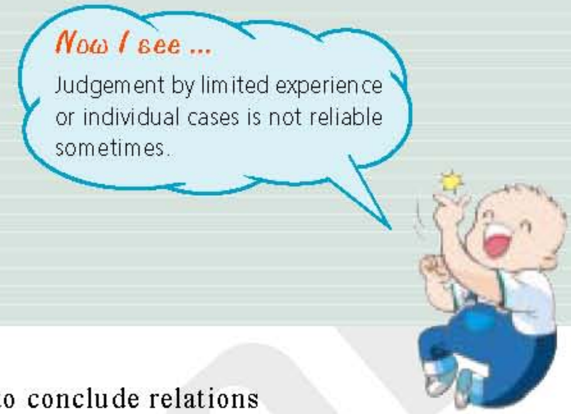


(a) Is the polygon equilateral?

☐ Yes ☐ No

(b) Is the polygon equiangular?

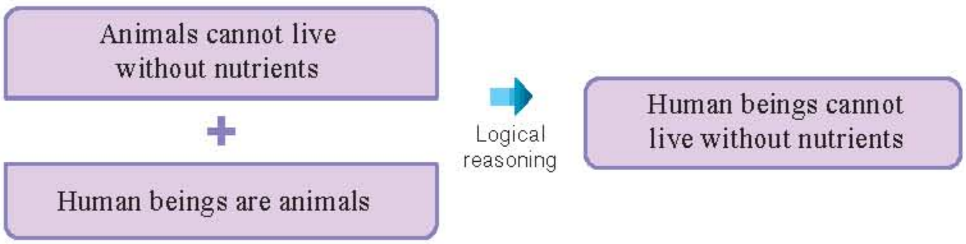
☐ Yes ☐ No



In Class Activities 13.5 and 13.6, we have tried to conclude relations among the objects by observation and experience. In fact, an **intuitive approach** has been used. However, as illustrated by these two class activities, this approach is not reliable sometimes.

B Deduction

We often make correct judgements through appropriate logical reasoning in daily life. The following chart shows the thinking process of the above.



intuitive approach 直觀法

e-BOOK

© 2009 Chung Tai Educational Press. All rights reserved.

To avoid getting incorrect knowledge through an intuitive approach, mathematicians always determine the correctness of knowledge through appropriate logical reasoning.

The following are examples of applying appropriate logical reasoning in mathematics.

Given information | | Conclusion

(a) Every prime number has only 2 factors

+

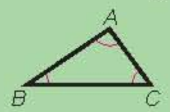
17 is a prime number

Logical reasoning → 17 has only 2 factors

(b) The sum of all interior angles of a triangle is 180°

+

ABC is a triangle



Logical reasoning → $\angle A + \angle B + \angle C = 180°$

In each of these examples, if all the given information is correct, the conclusion must be correct. This kind of reasoning is called **deductive reasoning**.

Correct condition(s) → Deductive reasoning → Correct conclusion

Based on correct condition(s), when a geometric fact is found to be true through deductive reasoning, we are proving the geometric fact by **deduction**.

e.g. In the figure, given $AB = CD$, prove that $AC = BD$.

Deductive reasoning

$$AB = CD$$
$$AB + BC = CD + BC$$
$$AC = BD$$

Deductive reasoning

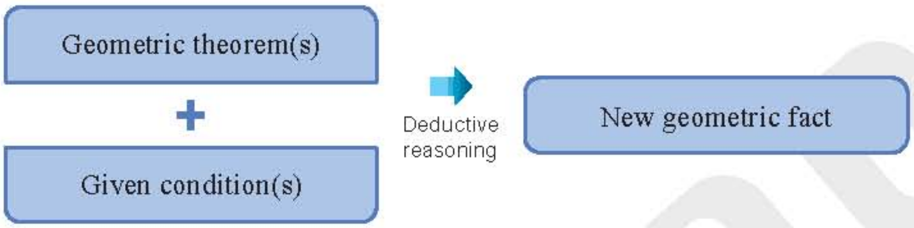
deductive reasoning 演繹推理 **deduction 演繹法**

© 2009 Chung Tai Educational Press. All rights reserved.
E-Book

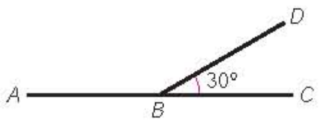
We can conclude that '$AB + BC = CD + BC$' with the correct condition '$AB = CD$'. Hence, we can draw the conclusion that '$AC = BD$' by treating '$AB + BC = CD + BC$' as another correct condition. Without any measuring tools, we can still ensure '$AC = BD$'.

In fact, the geometric properties we have learned can all be proved by deduction. However, due to the complexity of the process, their proofs are not discussed here. These proved properties are called geometric **theorems**.

Since all the geometric theorems we have learned are correct, we often make use of them together with some extra correct conditions to prove new geometric facts by deduction. The following is a common practice in geometric proofs.



e.g. In the figure, ABC is a straight line and $\angle CBD = 30°$.



Prove that $\angle ABD = 5\angle CBD$.

Proof: $\because$ ABC is a straight line (given)

'given' means a given condition

$$\begin{aligned} \therefore \quad \angle ABD + \angle CBD &= 180° \quad \text{(adj. } \angle\text{s on st. line)} \\ \angle ABD + 30° &= 180° \\ \angle ABD &= 150° \\ &= 5 \times 30° \\ &= 5\angle CBD \end{aligned}$$

In the above example, based on the given condition 'ABC is a straight line', we can make use of the theorem 'adj. $\angle$s on st. line' to conclude that '$\angle ABD + \angle CBD = 180°$'. Using this conclusion, '$\angle ABD = 5\angle CBD$' is deduced.

C Proofs related to straight lines and angles

In this section, we will learn some proofs related to straight lines and angles on a plane. The following theorem helps us determine a straight line by deduction.

theorem 定理

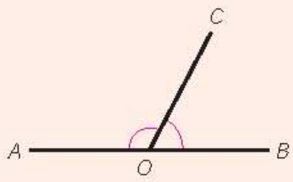
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

If the sum of two adjacent angles $\angle AOC$ and $\angle BOC$ is 180°, then AOB is a straight line.

i.e. If $\angle AOC + \angle BOC = 180°$,
then AOB is a straight line.

[Abbreviation: adj. ∠s supp.]



Notes: (a) The above theorem can be proved by deduction. Due to the complexity of the process, the proof is not discussed here.

(b) Two angles are supplementary to each other or two angles are supplementary angles means that the sum of the two angles is 180°.

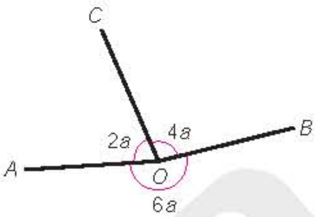
(c) 'adj. ∠s on st. line' and 'adj. ∠s supp.' are 2 different theorems.

Theorem	Given condition	Conclusion
adj. ∠s on st. line	AOB is a straight line	$\angle AOC + \angle BOC = 180°$
adj. ∠s supp.	$\angle AOC + \angle BOC = 180°$	AOB is a straight line

Additional Example 13.16

Example 13.16 Determining a straight line by deduction

In the figure, prove that AOB is a straight line.



Proof

[Analysis: If we know that '$\angle AOC + \angle BOC = 180°$', we can conclude that 'AOB is a straight line' by 'adj. ∠s supp.'. Since all the marked angles are at the same point, the proof can be started from this condition.]

$$2a + 4a + 6a = 360° \quad (\angle\text{s at a pt.})$$
$$12a = 360°$$
$$a = 30°$$

$$\because \quad \angle AOC + \angle BOC = 2a + 4a$$
$$= 6a$$
$$= 6 \times 30°$$
$$= 180°$$

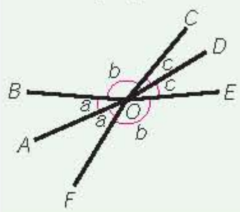
$\therefore$ AOB is a straight line. (adj. ∠s supp.)

Note: The geometric figures are not necessarily drawn to scale. For example, in the above figure, AOB does not look like a straight line.

Classwork 13.16

Solution

Prove that AOD is a straight line in the following figure.

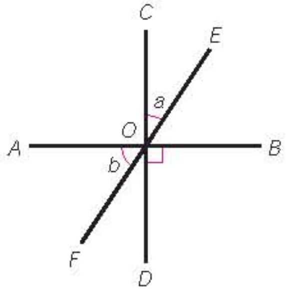


© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.17

Example 13.17 Proof for the relation among angles through the properties of straight lines

In the figure, AOB, COD and EOF are straight lines, $\angle BOD = 90°$. Prove that $a + b = 90°$.



Proof

[Analysis: Given that 'AOB, COD and EOF are straight lines', we start the proof by first considering the properties of these straight lines and make use of the related theorems.]

$\because$ AOB and COD are straight lines (given)

$\therefore$ $\angle AOC = \angle BOD$ (vert. opp. $\angle$s)

$= 90°$

$\because$ EOF is a straight line (given)

$\therefore$ $\angle COE + \angle AOC + \angle AOF = 180°$ (adj. $\angle$s on st. line)

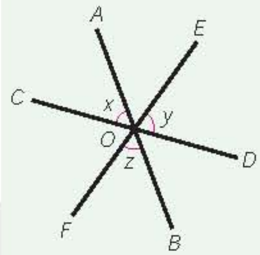
$a + 90° + b = 180°$

$\therefore$ $a + b = 180° - 90°$

$= 90°$

Classwork 13.17 Solution

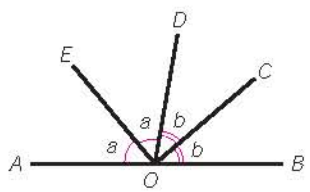
In the figure, AOB, COD and EOF are straight lines. Prove that $x + y + z = 180°$.



Additional Example 13.18

Example 13.18 Determining a right angle through the properties of straight lines

In the figure, AOB is a straight line, $\angle AOE = \angle EOD$ and $\angle DOC = \angle COB$. Prove that $\angle COE$ is a right angle.



Proof

$\because$ AOB is a straight line (given)

$\therefore$ $a + a + b + b = 180°$ (adj. $\angle$s on st. line)

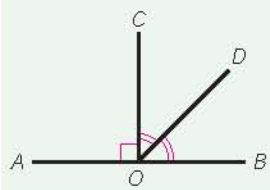
$2a + 2b = 180°$

$a + b = 90°$ ◂ Both sides are multiplied by $\frac{1}{2}$.

$\therefore$ $\angle COE$ is a right angle.

Classwork 13.18 Solution

In the figure, AOB is a straight line. $\angle AOC = 90°$ and $\angle COD = \angle DOB$. Prove that $\angle COD = 45°$.



e-BOOK

© 2009 Chung Tai Educational Press. All rights reserved.

D Determination of parallel lines

Is there any way to determine whether two straight lines are parallel? Do you still remember the method of drawing parallel lines by using set squares learned in Chapter 3 of S1A?

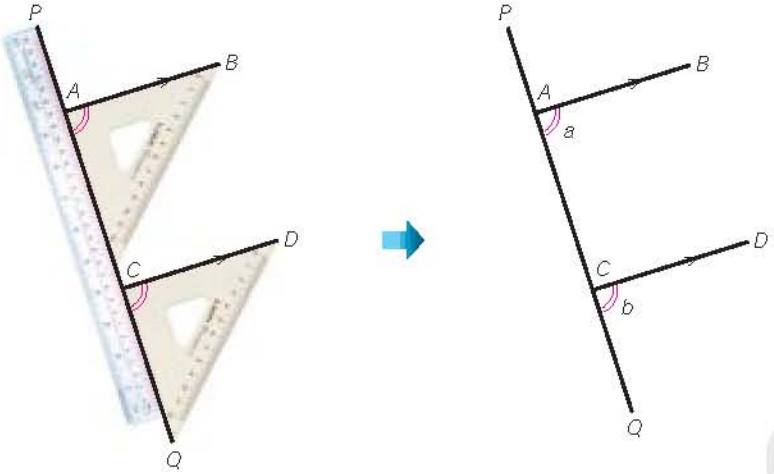
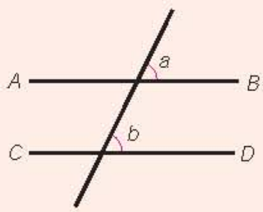


Figure 13.15

In fact, we have used the property of equal corresponding angles a and b as follows to obtain a pair of parallel lines AB and CD.

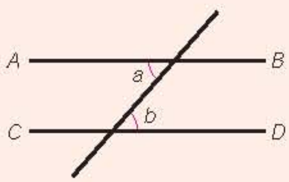
Two lines are parallel if corresponding angles made by a transversal are equal.
i.e. If $a = b$,
then $AB \parallel CD$.
[**Abbreviation:** corr. $\angle$s eq.]



Note: The above theorem can be proved by deduction. Due to the complexity of the process, the proof is not discussed here.

We can also use the properties of alternate angles and interior angles on the same side to determine whether two straight lines are parallel. The following are two related theorems where their proofs are based on the theorem 'corr. $\angle$s eq.'.

Two lines are parallel if alternate angles made by a transversal are equal.
i.e. If $a = b$,
then $AB \parallel CD$.
[**Abbreviation:** alt. $\angle$s eq.]



© 2009 Chung Tai Educational Press. All rights reserved.

Proof: $b = a$ (given)

$a = c$ (vert. opp. $\angle$s)

$\therefore$ $b = c$

$\therefore$ AB // CD (corr. $\angle$s eq.)

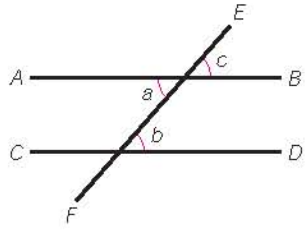
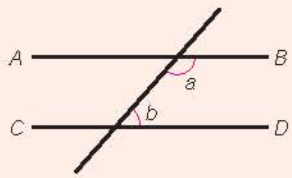


Figure 13.16

Two lines are parallel if interior angles on the same side made by a transversal are supplementary.
i.e. If $a + b = 180°$,
then AB // CD.
[**Abbreviation:** int. $\angle$s supp.]



Proof: $a + b = 180°$ (given)

$b = 180° - a$

$= c$ (adj. $\angle$s on st. line)

$\therefore$ AB // CD (corr. $\angle$s eq.)

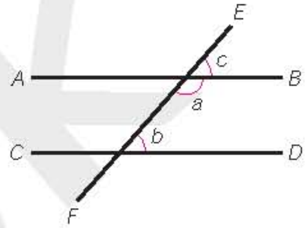
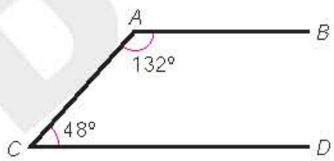


Figure 13.17

Additional Example 13.19

Example 13.19 Determination of parallel lines by deduction

In the figure, prove that AB // CD.



Proof

$\angle ACD + \angle CAB = 48° + 132°$

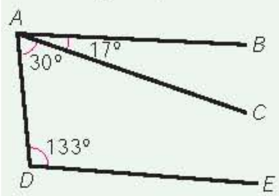
$= 180°$

$\therefore$ AB // CD (int. $\angle$s supp.)

Classwork 13.19

Solution

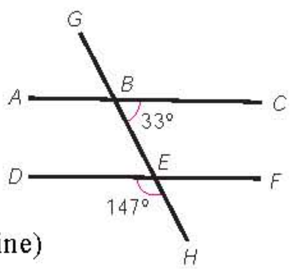
In the figure, prove that AB // DE.



Additional Example 13.20

Example 13.20 Determination of parallel lines through the properties of straight lines

In the figure, ABC, DEF and $GBEH$ are straight lines. Prove that AC // DF.



Proof

$\because$ $GBEH$ is a straight line (given)

$\therefore$ $\angle DEB = 180° - 147°$ (adj. $\angle$s on st. line)

$= 33°$

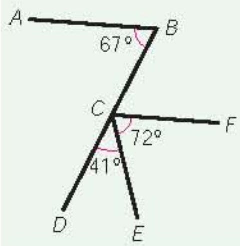
$= \angle CBE$

$\therefore$ AC // DF (alt. $\angle$s eq.)

Classwork 13.20

Solution

In the figure, BCD is a straight line. Prove that AB // CF.



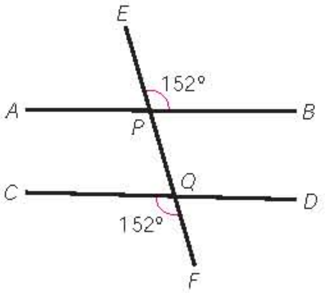
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.21

Example 13.21 Determination of parallel lines through the properties of straight lines

In the figure, APB, CQD and $EPQF$ are straight lines. Prove that $AB \parallel CD$.



Proof

$\because$ $EPQF$ and CQD are straight lines (given)

$\therefore$ $\angle PQD = \angle CQF$ (vert. opp. $\angle$s)

$= 152°$

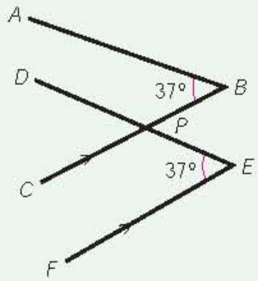
$= \angle EPB$

$\therefore$ $AB \parallel CD$ (corr. $\angle$s eq.)

Classwork 13.21

Solution

In the figure, DPE and BPC are straight lines and $BC \parallel EF$. Prove that $AB \parallel DE$.

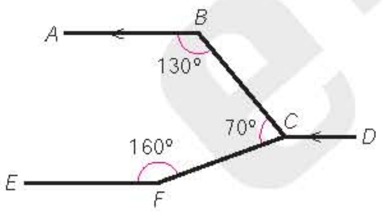


e-Book

Additional Example 13.22

Example 13.22 Determination of parallel lines through the properties of straight lines

In the figure, $AB \parallel CD$. Prove that $AB \parallel EF$.



Proof

$\because$ $AB \parallel CD$ (given)

$\therefore$ $\angle BCD = \angle ABC$ (alt. $\angle$s, $AB \parallel CD$)

$= 130°$

$\angle DCF + \angle BCF + \angle BCD = 360°$ ($\angle$s at a pt.)

$\angle DCF + 70° + 130° = 360°$

$\angle DCF = 160°$

$= \angle EFC$

$\therefore$ $CD \parallel EF$ (alt. $\angle$s eq.)

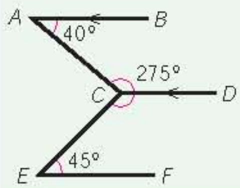
$\because$ $AB \parallel CD$ and $CD \parallel EF$

$\therefore$ $AB \parallel EF$

Classwork 13.22

Solution

In the figure, $AB \parallel CD$. Prove that $AB \parallel EF$.



© 2009 Chung Tai Educational Press. All rights reserved.

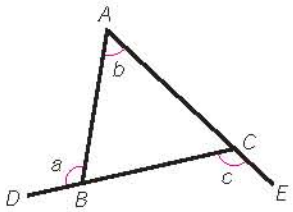
E Proofs related to triangles

Some geometric properties can be proved by '∠ sum of Δ', 'ext. ∠ of Δ' and 'base ∠s, isos. Δ', which have been learned in section 13.3.

Additional Example 13.23

Example 13.23 Proof for the relation among angles through the properties of triangles

In the figure, ACE and CBD are straight lines. Prove that $a - b + c = 180°$.



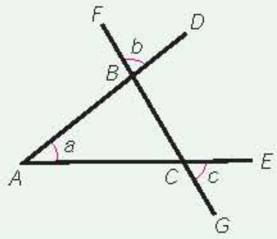
Proof

∵	ACE is a straight line	(given)
∴	$\angle ACB = 180° - c$	(adj. ∠s on st. line)
∵	CBD is a straight line	(given)
∴	$a = b + \angle ACB$	(ext. ∠ of Δ)
	$a = b + 180° - c$	
∴	$a - b + c = 180°$	

Classwork 13.23 Solution

In the figure, ABD, ACE and $FBCG$ are straight lines. Prove that

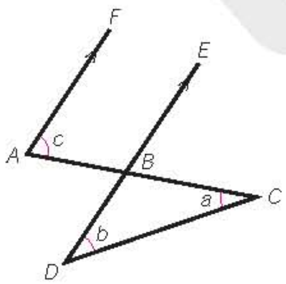
$a + b + c = 180°$.



Additional Example 13.24

Example 13.24 Proof for the relation among angles through the properties of parallel lines and triangles

In the figure, AF // DE, ABC and DBE are straight lines. Prove that $c = a + b$.

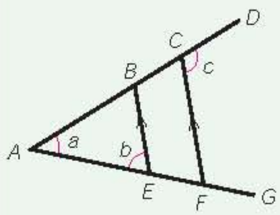


Proof

∵	ABC is a straight line	(given)
∴	$\angle ABD = a + b$	(ext. ∠ of Δ)
∵	AF // DE	(given)
∴	$c = \angle ABD$	(alt. ∠s, AF // DE)
	$= a + b$	

Classwork 13.24 Solution

In the figure, $ABCD$ and $AEFG$ are straight lines and BE // CF. Prove that $c = a + b$.

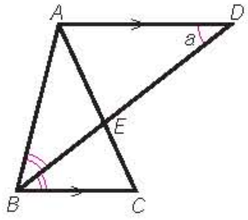


© 2009 Chung Tai Educational Press. All rights reserved.

Additional Example 13.25

Example 13.25 Proof for the relation among angles through the properties of parallel lines and isosceles triangles

In the figure, AEC and BED are straight lines. If $AB = AC$, $\angle ABD = \angle DBC$ and $AD \,//\, BC$, prove that $\angle ACB = 2a$.



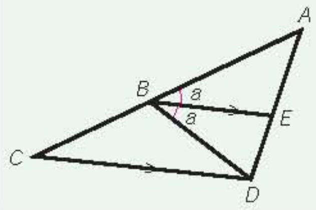
Proof

$\because$	BED is a straight line, and $AD \,//\, BC$	(given)
$\therefore$	$\angle EBC = a$	(alt. $\angle$s, $AD \,//\, BC$)
$\because$	$\angle ABE = \angle EBC$	(given)
$\therefore$	$\angle ABE = a$	
$\therefore$	$\angle ABC = a + a$	
	$= 2a$	
$\because$	$AB = AC$	(given)
$\therefore$	$\angle ACB = \angle ABC$	(base $\angle$s, isos. Δ)
	$= 2a$	

Classwork 13.25

Solution

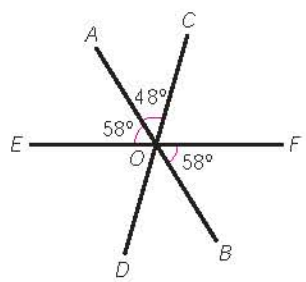
In the figure, ABC and AED are straight lines. $AB = AD$ and $BE \,//\, CD$. Prove that $\angle ADC = 3a$.



Skills Upgrading Corner 13.4

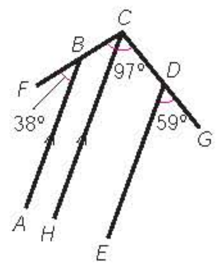
Solution

1. In the figure, AOB and COD are straight lines. Prove that EOF is a straight line.



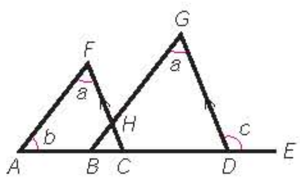
Solution

2. In the figure, CBF and CDG are straight lines and $AB \,//\, HC$.
   (a) Find $\angle HCG$.
   (b) Prove that $CH \,//\, DE$.



Solution

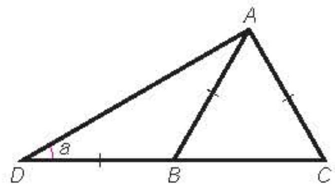
3. In the figure, $ABCDE$, BHG and CHF are straight lines and $FC \,//\, GD$.
   (a) Express c in terms of a and b.
   (b) Prove that $FA \,//\, GB$.



© 2009 Chung Tai Educational Press. All rights reserved.

Solution

4. In the figure, CBD is a straight line, $AB = BD = AC$ and $\angle ADB = a$. Prove that $\angle BAC = 180° - 4a$.

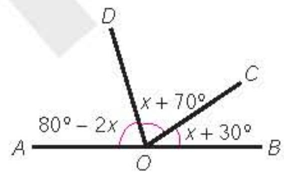


Exercise 13D

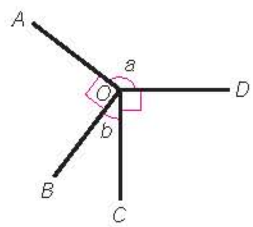
[The figures in this exercise are not necessarily drawn to scale.]

Level 1

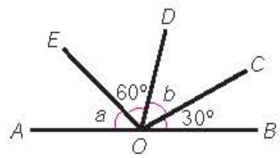
1. In the figure, prove that AOB is a straight line.



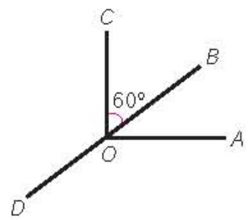
2. In the figure, $\angle AOB = \angle COD = 90°$. Prove that $a + b = 180°$.



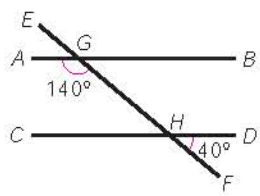
3. In the figure, AOB is a straight line. Prove that $a + b = 90°$.



4. In the figure, BOD is a straight line. If $\angle COD = 4\ \angle AOB$, prove that $CO \perp AO$.
[Hint: Prove that $\angle AOC = 90°$.]

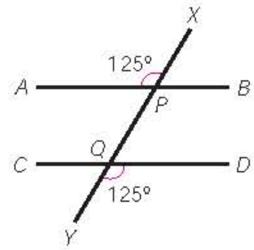


5. In the figure, AGB, CHD and $EGHF$ are straight lines. Prove that $AB \mathbin{/\!/} CD$.

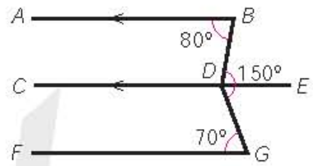


© 2009 Chung Tai Educational Press. All rights reserved.

6. In the figure, APB, CQD and $XPQY$ are straight lines. Prove that $AB \parallel CD$.



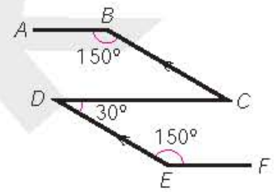
7. In the figure, CDE is a straight line, $AB \parallel CE$. Prove that $AB \parallel FG$.



8. In the figure, $BC \parallel DE$.

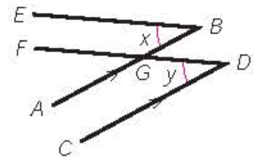
(a) Prove that $AB \parallel DC$.

(b) Prove that $AB \parallel EF$.

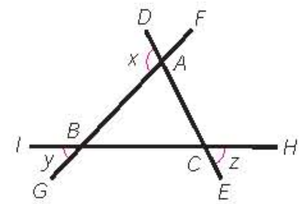


Level 2

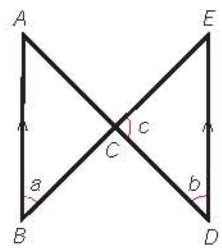
9. In the figure, AGB and FGD are straight lines, $AB \parallel CD$ and $x = y$. Prove that $EB \parallel FD$.



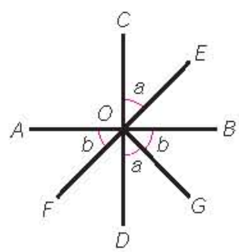
10. In the figure, $DACE$, $IBCH$ and $GBAF$ are straight lines. Prove that $x = y + z$.



11. In the figure, ACD and BCE are straight lines and $AB \parallel ED$. Prove that $a + b + c = 180°$.



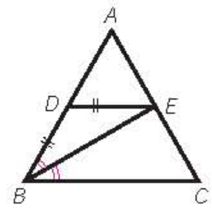
12. In the figure, AOB, COD and EOF are straight lines. Prove that $CD \perp AB$.



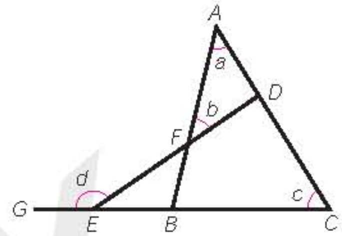
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

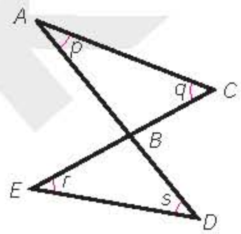
13. The figure shows ΔABC. It is known that $\angle ABE = \angle EBC$ and $DB = DE$. Prove that $DE \parallel BC$.



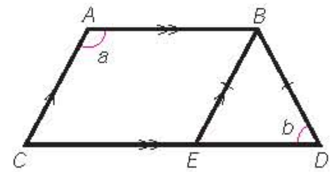
14. In the figure, ADC, AFB, $CBEG$ and DFE are straight lines. Prove that $d = a + b + c$.



15. In the figure, EC and AD intersect at B. Prove that $p + q = r + s$.



16. In the figure, CED is a straight line, $BE = BD$, $AC \parallel BE$ and $AB \parallel CD$. Prove that $a + b = 180°$.

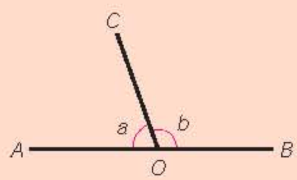


Chapter Summary

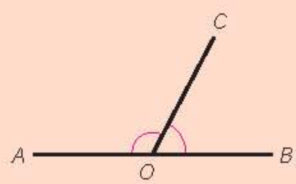
Chapter Summary

Fact to Remember

1. The sum of adjacent angles on a straight line is 180°.
i.e. If AOB is a straight line,
then $a + b = 180°$.
[**Abbreviation:** adj. ∠s on st. line]



2. If the sum of two adjacent angles $\angle AOC$ and $\angle BOC$ is 180°, then AOB is a straight line.
i.e. If $\angle AOC + \angle BOC = 180°$,
then AOB is a straight line.
[**Abbreviation:** adj. ∠s supp.]



e-Book

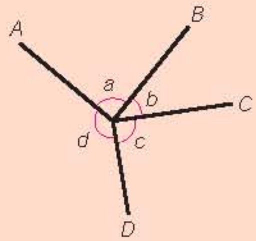
© 2009 Chung Tai Educational Press. All rights reserved.

3. The sum of angles at a point is 360°.

i.e. If a, b, c and d are angles at a point,

then $a + b + c + d = 360°$.

[**Abbreviation:** ∠s at a pt.]

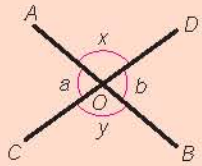


4. Vertically opposite angles are equal.

i.e. If AOB and COD are straight lines,

then $a = b$ and $x = y$.

[**Abbreviation:** vert. opp. ∠s]

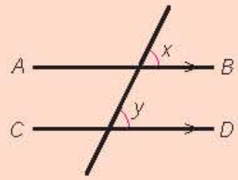


5. If a transversal cuts a pair of parallel lines, then the corresponding angles are equal.

i.e. If AB // CD,

then $x = y$.

[**Abbreviation:** corr. ∠s, AB // CD]

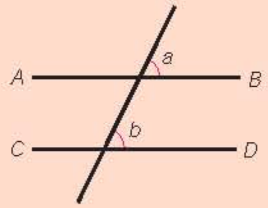


6. Two lines are parallel if corresponding angles made by a transversal are equal.

i.e. If $a = b$,

then AB // CD.

[**Abbreviation:** corr. ∠s eq.]

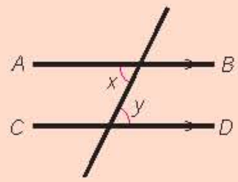


7. If a transversal cuts a pair of parallel lines, then the alternate angles are equal.

i.e. If AB // CD,

then $x = y$.

[**Abbreviation:** alt. ∠s, AB // CD]

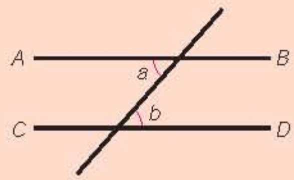


8. Two lines are parallel if alternate angles made by a transversal are equal.

i.e. If $a = b$,

then AB // CD.

[**Abbreviation:** alt. ∠s eq.]

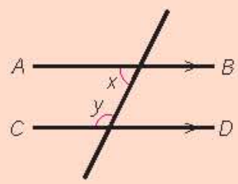


9. If a transversal cuts a pair of parallel lines, then the sum of interior angles on the same side is 180°.

i.e. If AB // CD,

then $x + y = 180°$.

[**Abbreviation:** int. ∠s, AB // CD]



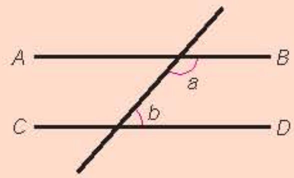
© 2009 Chung Tai Educational Press. All rights reserved.

10. Two lines are parallel if interior angles on the same side made by a transversal are supplementary.

i.e. If $a + b = 180°$,

then $AB \parallel CD$.

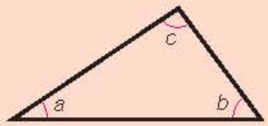
[**Abbreviation:** int. ∠s supp.]



11. The sum of all interior angles of a triangle is 180°.

i.e. $a + b + c = 180°$.

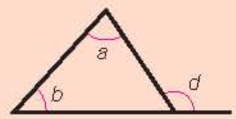
[**Abbreviation:** ∠ sum of Δ]



12. The exterior angle of a triangle is equal to the sum of its two interior opposite angles.

i.e. $d = a + b$.

[**Abbreviation:** ext. ∠ of Δ]

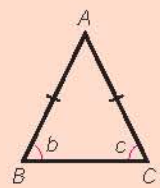


13. The base angles of an isosceles triangle are equal.

i.e. If $AB = AC$,

then $b = c$.

[**Abbreviation:** base ∠s, isos. Δ]

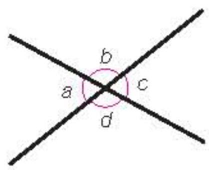


Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

1. **(a)** In the figure, a, b, c and d are four angles formed at the intersection of two straight lines. 13.1

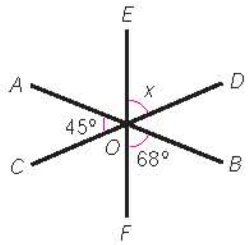


a and b are adjacent angles / vertically opposite angles.

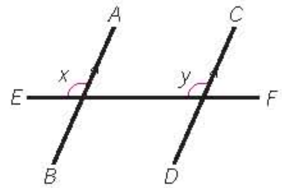
a and c are adjacent angles / vertically opposite angles.

© 2009 Chung Tai Educational Press. All rights reserved.

(b) In the figure, AOB, COD and EOF are straight lines. Find x.

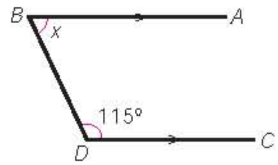


2. **(a)** In the figure, EF is a transversal of parallel lines AB and CD. 13.2

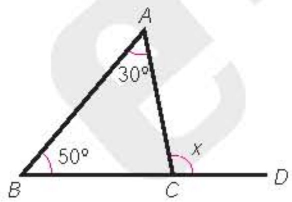


x and y is a pair of corresponding angles / alternate angles / interior angles on the same side.

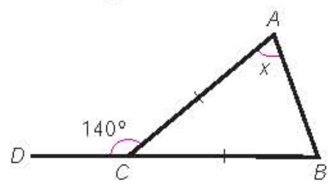
(b) In the figure, AB // CD. Find x.



3. **(a)** In the figure, if BCD is a straight line, then $x =$ __________ . 13.3



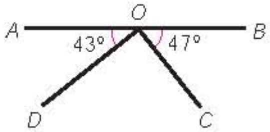
(b) In the figure, BCD is a straight line. Find x.



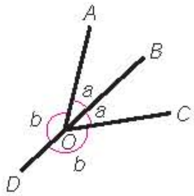
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

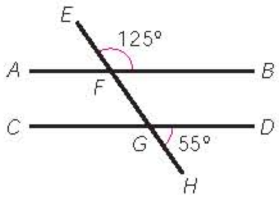
4. **(a)** In the figure, AOB is a straight line. Prove that $\angle COD$ is a right angle. 13.4C



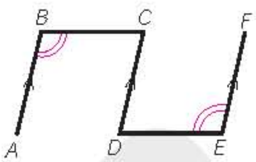
(b) In the figure, prove that BOD is a straight line.



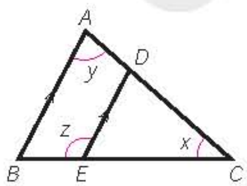
5. **(a)** In the figure, AFB, CGD and $EFGH$ are straight lines. Prove that AB // CD. 13.4D



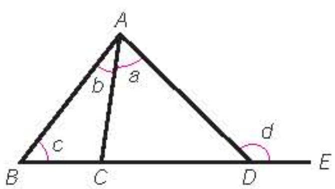
(b) In the figure, AB // DC // EF and $\angle ABC = \angle DEF$. Prove that BC // DE.



6. **(a)** In the figure, ADC and BEC are straight lines, AB // DE. Prove that $x + y = z$. 13.4E



(b) In the figure, $BCDE$ is a straight line. Prove that $a + b + c = d$.



e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

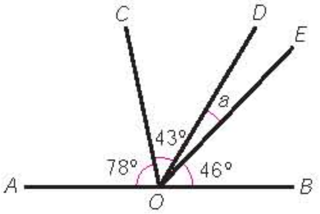
Revision Exercise 13

[The figures in this exercise are not necessarily drawn to scale.]

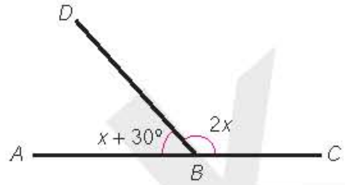
Level 1

Find the unknowns in each of the following figures. (**1** – **10**)

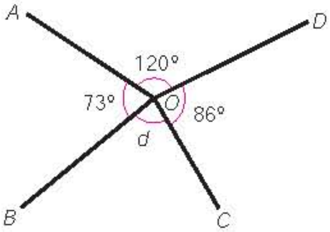
1. AOB is a straight line.



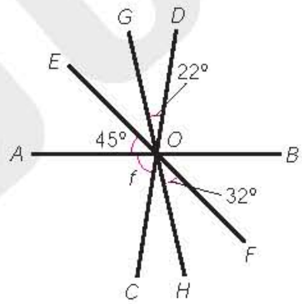
2. ABC is a straight line.



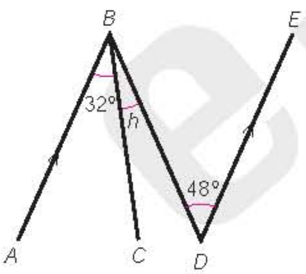
3.



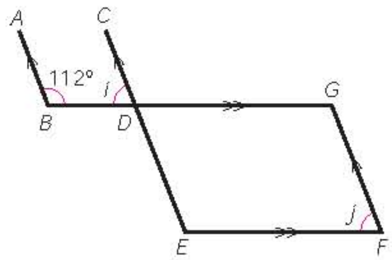
4. AOB, COD, EOF and GOH are straight lines.



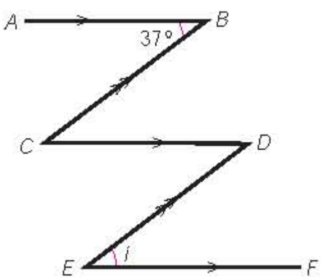
5.



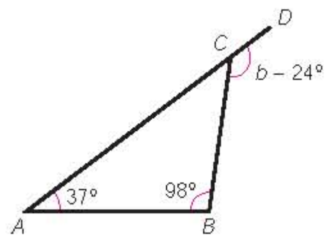
6. BDG and CDE are straight lines.



7.



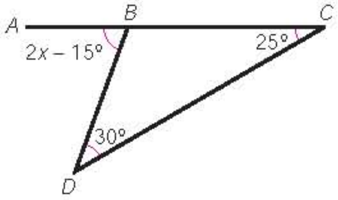
8. ACD is a straight line.



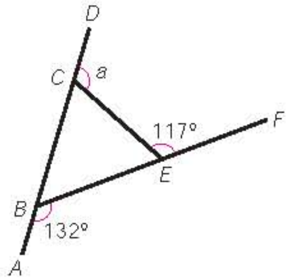
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

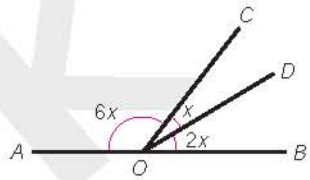
9. ABC is a straight line.



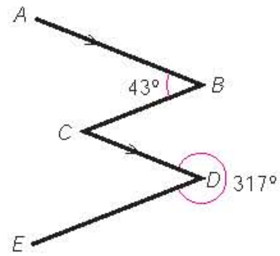
10. $ABCD$ and BEF are straight lines.



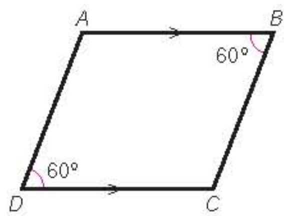
11. In the figure, if $x = 20°$, prove that AOB is a straight line.



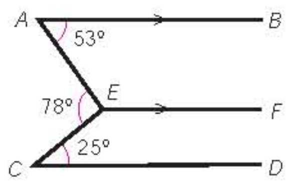
12. In the figure, $AB \parallel CD$. Prove that $CB \parallel ED$.



13. In the figure, $AB \parallel DC$ and $\angle ADC = \angle ABC = 60°$. Prove that $AD \parallel BC$.



14. In the figure, $AB \parallel EF$. Prove that $AB \parallel CD$.



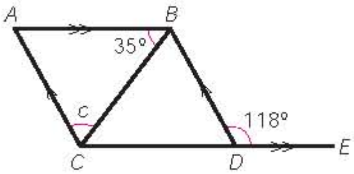
e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

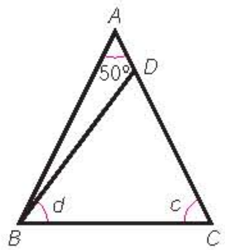
Level 2

Find the unknowns in each of the following figures. (**15 – 23**)

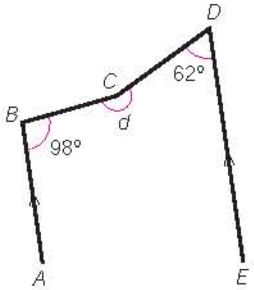
15. CDE is a straight line.



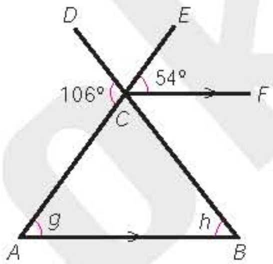
16. ADC is a straight line, $AB = AC$ and $BD = BC$.



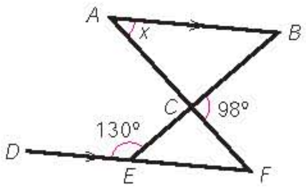
17.



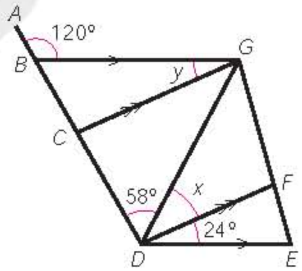
18. DCB and ECA are straight lines.



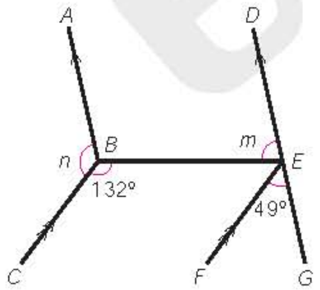
19. ACF, BCE and DEF are straight lines.



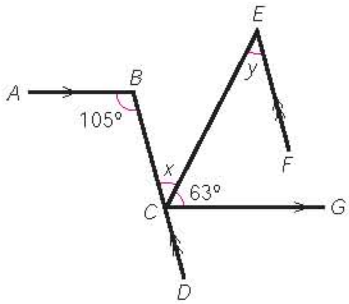
20. $ABCD$ and GFE are straight lines.



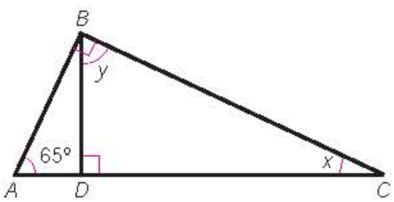
21. DEG is a straight line.



22. BCD is a straight line.



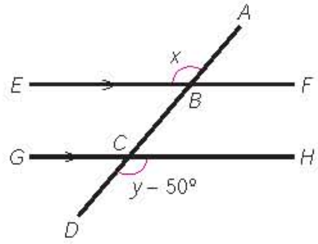
23. ADC is a straight line.



e-BOOK

© 2009 Chung Tai Educational Press. All rights reserved.

24. In the figure, AD intersects EF and GH at B and C respectively, EF // GH. Write down two possible sets of values of x and y. Explain your answer.



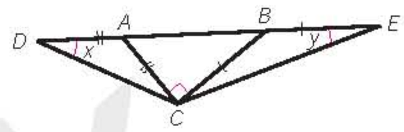
25. In the figure, $DABE$ is a straight line. $AC = AD$, $BC = BE$ and $\angle ACB = 90°$.

(a) Express $\angle CAB$ in terms of x.

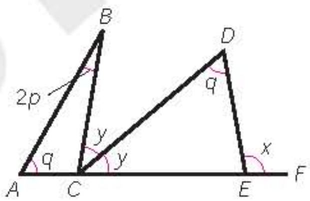
(b) Express $\angle CBA$ in terms of y.

(c) Find $x + y$.

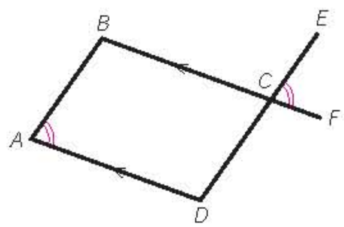
(d) Find $\angle DCE$.



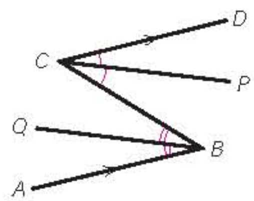
26. In the figure, $ACEF$ is a straight line. Prove that $x + y = 2(p + q)$.



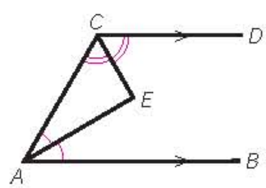
27. In the figure, BCF and DCE are straight lines, AD // BF and $\angle BAD = \angle ECF$. Prove that AB // DE.



28. In the figure, $\angle DCP = \angle BCP$, $\angle CBQ = \angle ABQ$ and AB // CD. Prove that QB // CP.



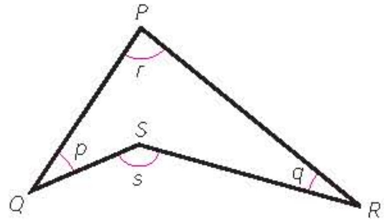
29. In the figure, $\angle CAE = \angle BAE$, $\angle ECD = \angle ECA$ and AB // CD. Prove that $\angle CEA$ is a right angle.



e-Book

© 2009 Chung Tai Educational Press. All rights reserved.

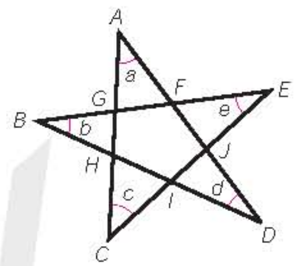
30. In the figure, prove that $s = p + q + r$.



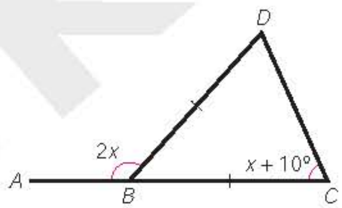
31. In the figure, $AGHC$, $AFJD$, $BGFE$, $BHID$ and $CIJE$ are straight lines.

(a) Prove that $a + c = \angle FJE$.

(b) Prove that $a + b + c + d + e = 180°$.



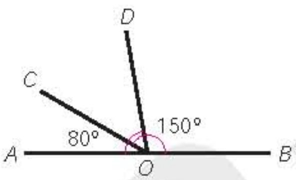
32. In the figure, it is known that ABC is a straight line and $BC = BD$. However, there are something unreasonable about the angles. Find them out and correct them.



e-Book

MC Question

33. In the figure, AOB is a straight line. Find $\angle COD$.



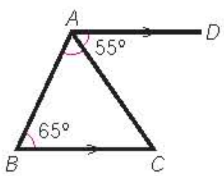
A. 30°

B. 40°

C. 50°

D. 60°

34. In the figure, $AD // BC$. $\angle BAC =$



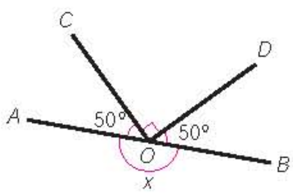
A. 50°.

B. 55°.

C. 60°.

D. 65°.

35. In the figure, $x =$



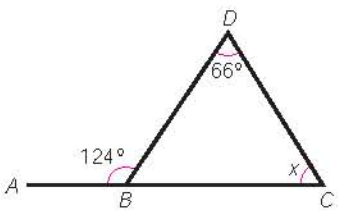
A. 170°.

B. 180°.

C. 190°.

D. 200°.

36. In the figure, ABC is a straight line. Find x.



A. 56°

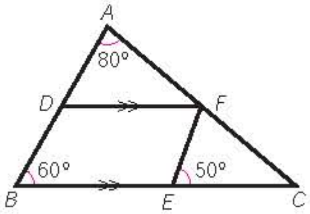
B. 58°

C. 62°

D. 66°

© 2009 Chung Tai Educational Press. All rights reserved.

37. In the figure, ADB, BEC and AFC are straight lines and $DF \parallel BC$.



Which of the following must be correct?

I. $\angle ADF = 60°$

II. $\angle CFE = 80°$

III. $\angle ACB = 40°$

A. I and II only

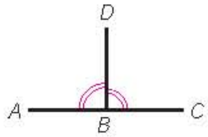
B. II and III only

C. I and III only

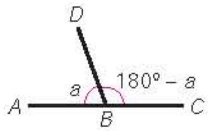
D. I, II and III

38. In which of following figures, ABC must be a straight line?

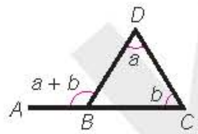
I.



II.



III.



A. I and II only

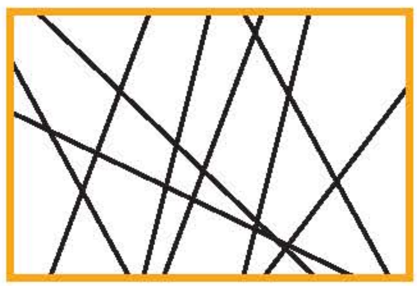
B. I and III only

C. II and III only

D. I, II and III

Problem-solving and Exploring

Hint for the Title Page Question



(a) Each of the following cases can help us find the parallel lines in the figure. For each case, briefly explain your method.

(i) Having a piece of transparent paper with the figure copied on it.

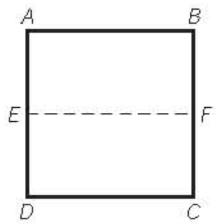
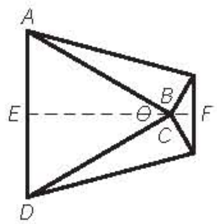
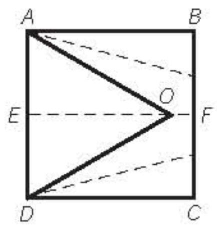
(ii) Having a pair of set squares.

(b) Can you think of any other method?

© 2009 Chung Tai Educational Press. All rights reserved.

Additional Question

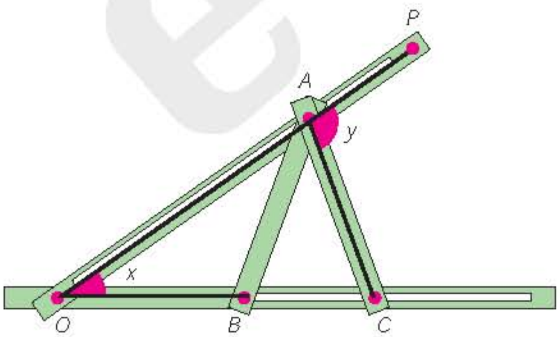
1. Get a piece of square paper, fold it according to the following steps.

Step 1	Fold the square paper $ABCD$ in half such that a crease EF is obtained.	
Step 2	Fold up the paper as shown in the figure such that B and C overlap with point O lying on the crease EF.	
Step 3	Unfold the paper, join OA and OD. Then ΔOAD is obtained.	

(a) What type of triangle is ΔOAD?

(b) Explain the result in **(a)**.

2. Michael has designed an instrument which can draw one third of an angle (see the figure below).



By adjusting the position of OP to make y the given angle, x will be equal to $\frac{1}{3}y$. The key to his design is $OB = AB = AC$. Based on your geometrical knowledge, prove that $x = \frac{1}{3}y$.

© 2009 Chung Tai Educational Press. All rights reserved.